USPAS Graduate Accelerator Physics Homework 9

Due date: Friday February 5, 2021

1 Tune plane resonances

(10 points) Consider a unit square in the tune plane (Q_x, Q_y) with corners at (n, n), $(n+1, n)$, $(n, n+1)$, and $(n+1, n+1)$.

a) On graph paper or with a computer program, draw the lines representing all sum resonances $p = q\,Q_x + r\,Q_y$ through fourth order – for positive integer values of q and r, with $q + r \leq 4$.

b) Plot all difference resonances $p = q\,Q_x - r\,Q_y$ through fourth order.

c) Where are the largest areas of tune space that are resonance-free?

2 Closed three-bumps

(10 points) The trigonometric law of sines states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \tag{2.1}$$

where A, B, and C are the angles of a triangle, while a, b, and c are the lengths of the opposing sides.

a) Use the law of sines to show that Equation 8.17 guarantees the localisation of a three-bump.

b) What are the ratios of corrector strengths that close the three-bump if the phase advance between neighbouring correctors is 60 degrees, or 90 degrees?

c) What phase advance conditions make three-bump localisation difficult in practice? Why?

3 Interaction region quadrupole strength errors

(10 points) The interaction region quadrupole Q2 in RHIC has a focal length of about 3.0 m, at a location where the β-function is about 1400 m in collision optics with $\beta^* = 1$ m.

a) How accurately must the strength of this magnet be known and set, if the strength error must be guaranteed to generate a β-wave amplitude of less than 1%?

b) What tune shift is generated at this level of error?