USPAS Accelerator Physics 2021
Texas A&M University

Introduction to Superconducting RF

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http://www.toddsatogata.net/2021-USPAS

Overview

- Recap
- Cavity Parameters
- Exercise I
- DC Superconductivity
- Exercise II
- RF Superconductivity
- Using Cavities
- Things to Remember

R E C A P

C: PILLBOXES — CYLINDRICAL RESONANT CAVITIES

Add flat ends at $z=0$ & $z=L$

Take a pair of waveguide modes with

$$k = \pm k_p = \pm p\frac{\pi}{L}, \quad p = 0, 1 \ldots \infty$$

add together ($\sim e^{\pm ikz}$) to get a resonant TM mode

$$E_z = \psi(r,\theta) \cdot \cos\left(p\pi\frac{z}{L}\right) \cdot e^{i\omega_{RES} t}$$

R E C A P

TM$_{0n0}$ mode has ONLY 2 non-zero field components

$$E_z = E_0 * J_0\left(u_{0n}\frac{r}{R}\right)e^{-i\omega_{0n0}t}$$

$$B_\theta = -B_0 \frac{\omega_{0n0} R}{u_{0n} c} * J_1\left(u_{0n}\frac{r}{R}\right)\cdot e^{-i\omega_{0n0}t}$$

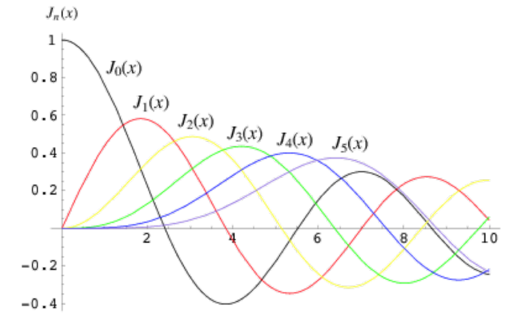
See Figure 7.4 for TM$_{020}$ .....

"n" counts the E_z zero-crossings from $r=0$ to R

peggs@bnl.gov Accelerator Physics, USPAS 2021 12

TM mnp

- m is order of Bessel function in Ez
- m = 0 implies no theta dependence.



R E C A P

It aquires a voltage of

$$V_A = \int_{-L/2}^{+L/2} E_z \, dz = \beta c . E_0 \int_{-L/2\beta c}^{+L/2\beta c} e^{i\omega t} . dt$$

For a TM_{010} mode

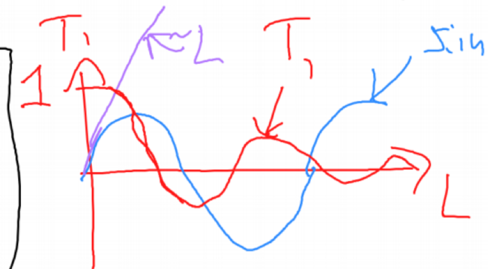
NOTE

$$V_A = E_0 L . T(L)$$

— TRANSIT TIME FACTOR

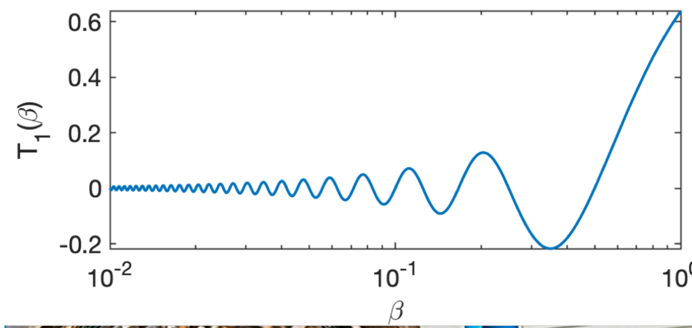
where

$$T_1(L) = \frac{\sin(\omega L/2\beta c)}{\omega L/2\beta c}$$



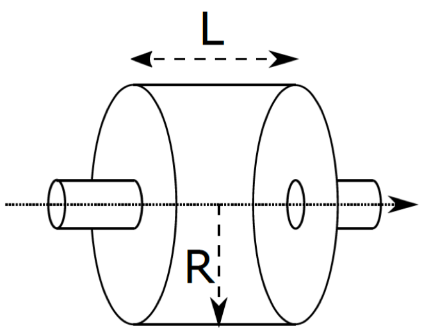
Realistic Cavities: High Beta

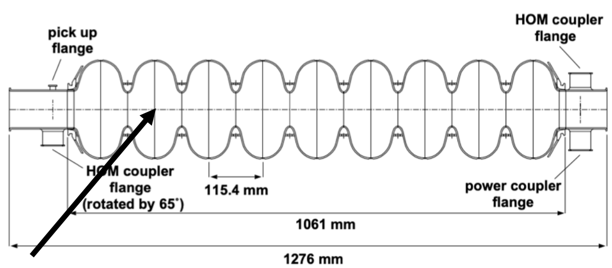
High Beta Cavities



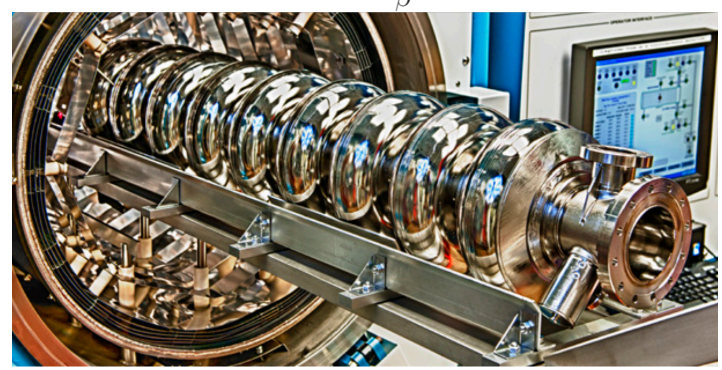
Particles can stay in phase with the with the field.

For a pillbox: $T_1(\beta) = \dfrac{\sin(\omega L/(2\beta c))}{\omega L/(2\beta c)} = \dfrac{\sin(\pi L/(\beta\lambda_{\mathrm{rf}}))}{\pi L/(\beta\lambda_{\mathrm{rf}})}$





Cells TESLA (Most electron linacs)

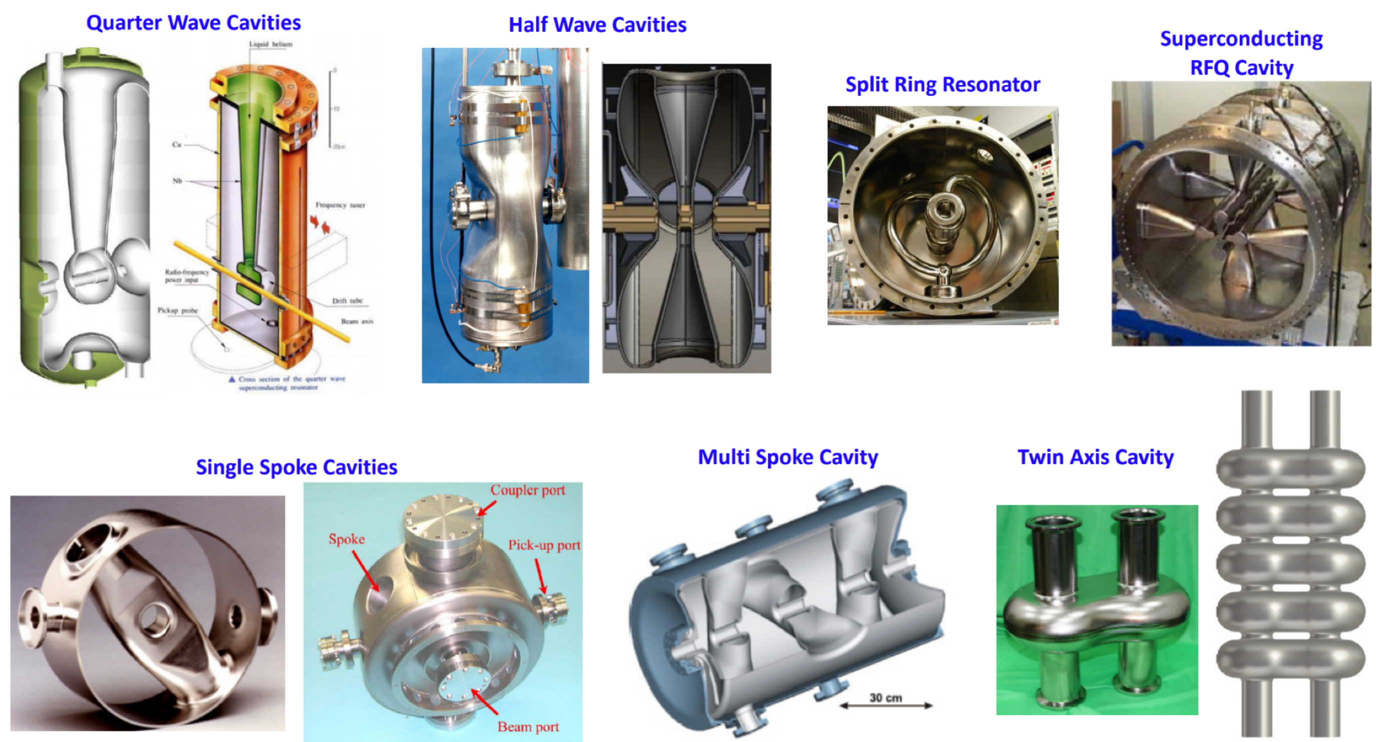


https://news.fnal.gov/2015/11/fermilab-attains-unprecedented-quality-factor-for-lcls-ii-accelerator-cavity/

Realistic Cavities: Other Designs

Many designs!

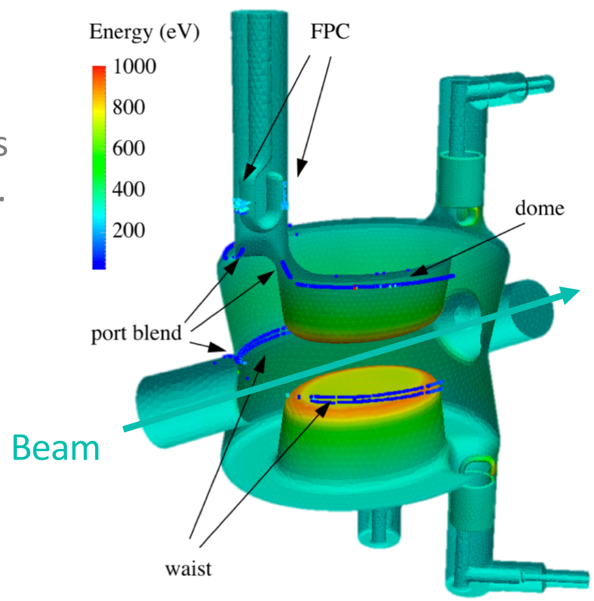
S.U. De Silva, “Superconducting Cavities of Interesting Shapes (Non-Elliptical Cavities)”, SRF'19, FRTU1. http://accelconf.web.cern.ch/srf2019/talks/frtu1_talk.pdf

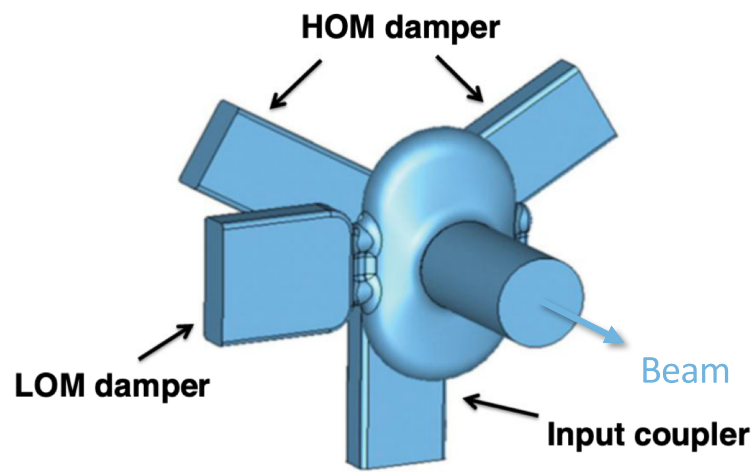


Even more

Cavities are used to ‘crab’ and deflect beams.

S. Verdú-Andrés et al., Phys. Rev. Accel. Beams 21, 082002, 2018





Yawei Yang et al., Phys. Rev. ST Accel. Beams 17, 032001, 2014

We need generalized parameters to quantify the performance.

Accelerating Voltage

The energy gained by an ultra-relativistic particle when it passes through a cavity.

$$\Delta\mathcal{E}(\phi) = q\int_0^L E_{z_0}(r=0,z)\cos\left(\frac{\omega_0 z}{c}+\phi\right)\mathrm{d}z$$

Particle entry phase **Fundamental mode field map**

Accelerating voltage (in Volts) is defined as the maximum energy gain per unit charge of an ultra-relativistic particle travelling through a resonant cavity.

$$V_c = \max_\phi \frac{\Delta\mathcal{E}(\phi)}{q} = \left|\int_0^L E_{z_0}(r=0,z)\mathrm{e}^{\frac{i\omega_0 z}{c}}\,\mathrm{d}z\right|$$

Stored Energy and Dissipation

The **total energy** stored in the electromagnetic field of the fundamental mode is

$$U = \frac{1}{2}\epsilon_0 \int_V |\vec{E}(\vec{r})|^2 \,\mathrm{d}V = \frac{1}{2}\mu_0 \int_V |\vec{H}(\vec{r})|^2 \,\mathrm{d}V$$

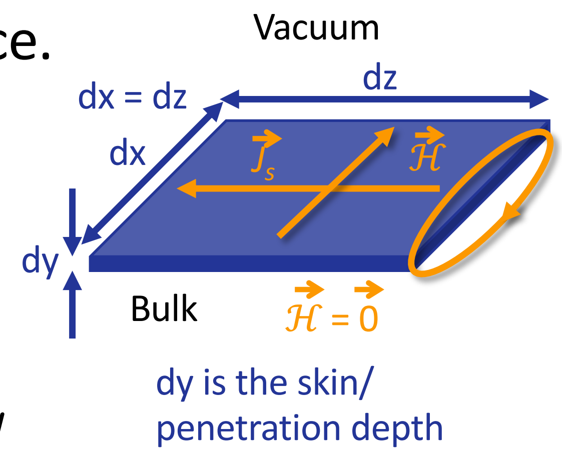
The total **power dissipated** on the cavity wall is *by definition*

$$P_{\mathrm{Wall}} = \frac{\omega U}{\boxed{Q_0}}$$

Intrinsic Quality Factor

Surface Resistance

Consider an infinitesimal *square* patch on the cavity surface.



Current $\vec{J}_s$ flows along the surface to generate the eigen mode magnetic field $\vec{\mathcal{H}}$. At the surface, the eigenmode electric field parallel to the surface is 0. Writing the Maxwell's equation for this patch of the surface.

No tangential electric field at surface.

$$\vec{\nabla} \times \vec{\mathcal{H}} - \epsilon_0 \frac{\partial \vec{\mathcal{E}}}{\partial t} = \vec{J}_s \implies |\vec{\mathcal{H}}|\,\mathrm{d}x = |\vec{J}_s|\,\mathrm{d}x\,\mathrm{d}y$$

Only works for an eigenmode!

The heat dissipated by the patch of surface is,

$$\mathrm{d}P_{\mathrm{Wall}} = I^2 R_s = (|\vec{J}_s|\,\mathrm{d}x\,\mathrm{d}y)^2 R_s = R_s |\vec{\mathcal{H}}|^2 \mathrm{d}x\,\mathrm{d}z$$

$dx^2 = dx\ dz$

R_s is called the **surface resistance**.

Dissipation and Geometry Factor

We can assume that the eigenmode magnetic field is $\vec{\mathcal{H}}(\vec{r}, t) = \vec{H}(\vec{r}) \exp(i\omega_0 t)$

Then total **power dissipated** on the cavity surface is,

$$P_{\text{Wall}} = \frac{R_s}{2} \int_S |\vec{H}(\vec{r})|^2 \, \mathrm{d}S$$

Factor of 2 from averaging.

The **geometry factor (in Ohms)** is defined as,

$$G \equiv Q_0 R_s = \omega_0 \mu_0 \frac{\int_V |\vec{H}|^2 \, \mathrm{d}V}{\int_S |\vec{H}|^2 \, \mathrm{d}S}$$

G is independent of cavity size, material and only dependent on shape!

Shunt Impedance

The **shunt impedance** of the cavity relates the accelerating voltage to the power dissipated while operating the cavity and is given by,

Circuit Definition

$$R \equiv \frac{V_c^2}{2P}$$

V_c is the accelerating voltage.

P is the *total* power lost during operation which includes heat dissipation on the wall and losses at the coupler.

A more useful quantity is the **ratio between the shunt impedance and a quality factor.**

Circuit Definition

$$\frac{R}{Q} = \frac{\frac{V_c^2}{2P}}{\frac{\omega_0 U}{P}} = \frac{V_c^2}{2\omega_0 U}$$

Quantifies the coupling between the beam and the resonant mode.

R/Q is independent of material properties!

Exercise I

Now we know two *Figures of Merit* of a rf cavity which are independent of material properties.

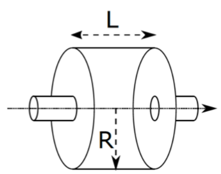
$$G \equiv Q_0 R_s = \omega_0 \mu_0 \frac{\int_V |\vec{H}|^2 \, \mathrm{d}V}{\int_S |\vec{H}|^2 \, \mathrm{d}S}$$

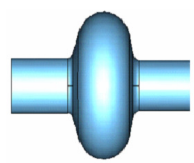
$$\frac{R}{Q} = \frac{\frac{V_c^2}{2P}}{\frac{\omega_0 U}{P}} = \frac{V_c^2}{2\omega_0 U}$$

While designing a cavity, which among these two quantities should we try to maximize or minimize?

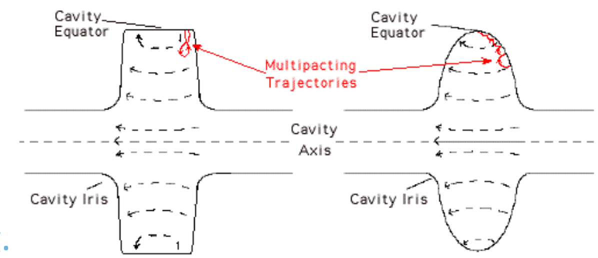
Figures of Merit: Summary

Cavity Parameters – Calculated using EM simulation codes.





Curved shape to stop multipacting.



Parameter	Pillbox	TESLA	Comment
R/Q (Ω)	98	57.55/cell	Larger value implies more acceleration with same rf power input.
G (Ω)	256.6	270	Larger value implies larger Q_0 with same R_s.
Epk/Eacc	1.6	2.0	Smaller values allow smaller surface fields for a fixed voltage.
Bpk/Eacc (mT/(MV/m))	3.05	4.26	

TESLA: https://arxiv.org/pdf/physics/0003011.pdf

The case for SRF

Material Property – Measured through rf testing.

Cryogenic copper: 250 MV/m @ 45 K!
https://journals.aps.org/prab/abstract/10.1103/PhysRevAccelBeams.21.102002

Material	Copper (300 K)	Copper (2 K)	Niobium (2 K)
Surface Resistance @ 1.3 GHz	10 mΩ	2 mΩ	10 nΩ
Dissipation in a Pillbox @ 1 MV	200 kW	40 kW	**0.2 W**

Pulsed: 1%→2 kW

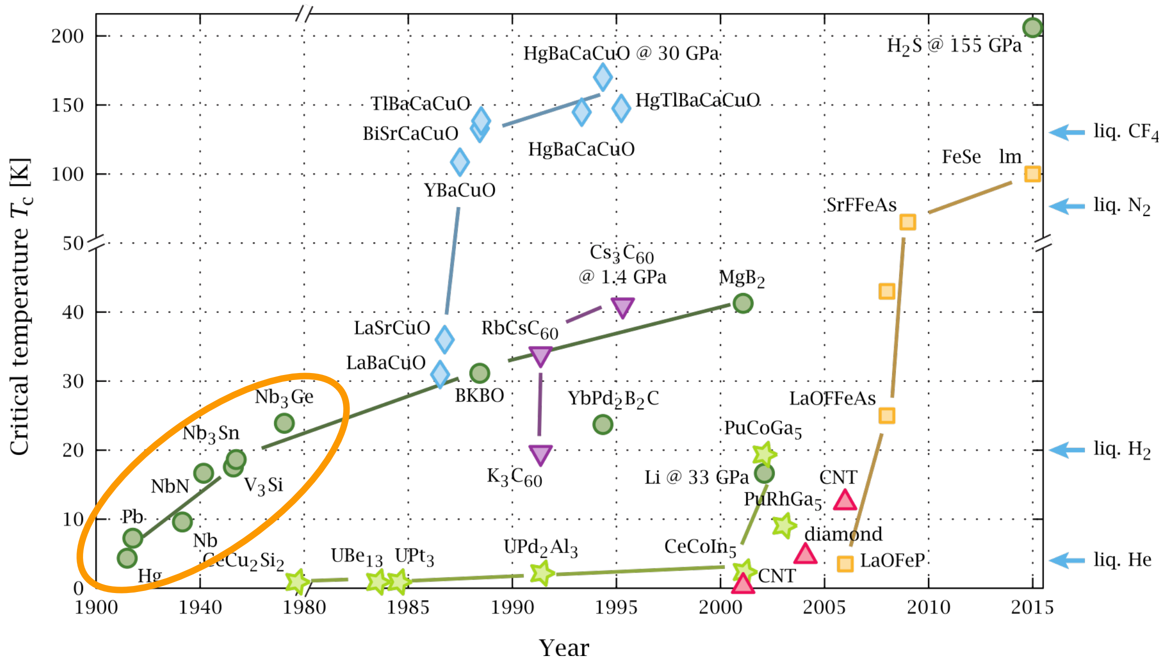
Case for SRF?

But cryogenic systems are power hungry! Coefficient of Performance: $COP \equiv \frac{q}{W} \sim 0.001 \,@\, 2\,\mathrm{K}$

The net reduction in power consumption is typically a factor of 100 when using SRF.
SRF is necessary when dealing with CW systems.
But why do superconducting materials exhibit a resistance under rf excitation?

DC Superconductivity

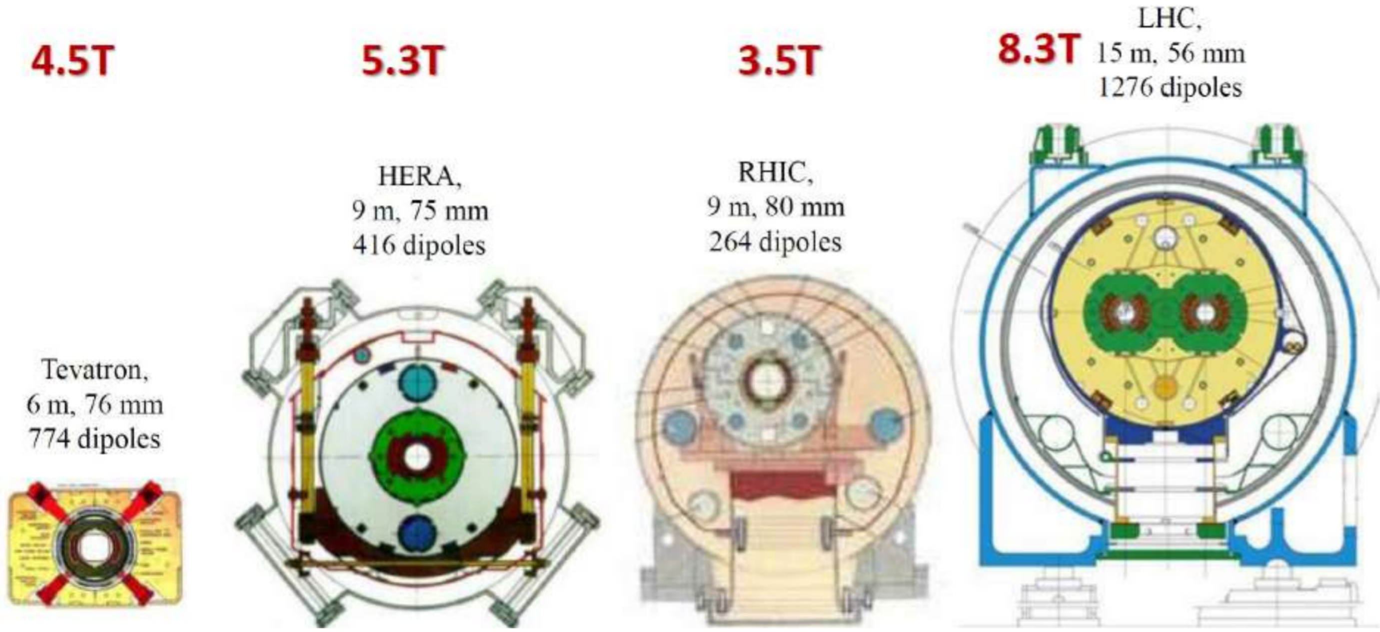
Zero electrical resistance below a **critical temperature**. First discovered in solid mercury by Heike Kamerlingh Onnes in 1911.



By PJRay - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=46193149

Only interested in BCS superconductors.

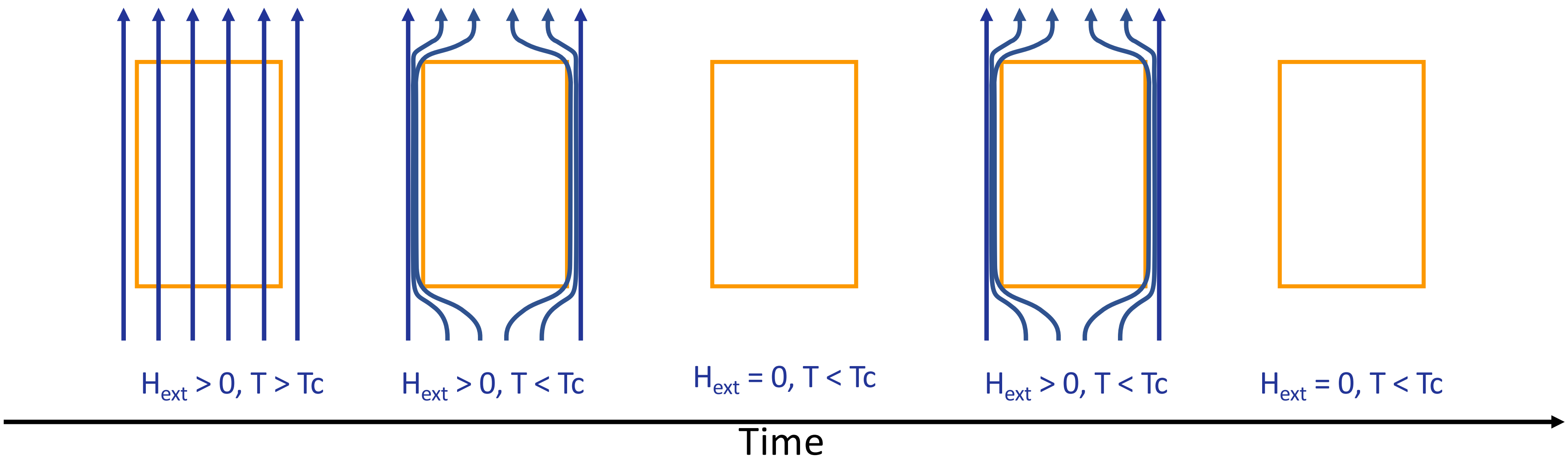
Superconducting Magnets



V Shiltsev, Accelerator science and technology breakthroughs, achievements and lessons from the Tevatron, arxiv, https://arxiv.org/pdf/1109.1864.pdf

Meissner Effect

A superconductor expels ***all*** magnetic field from the bulk just below transition.



Magnetic field inside the superconductor is 0 independent of the history of the external applied magnetic field.

London Equations

Consider superconducting charged carriers which move without getting scattered.

$$\vec{j}_s = n_s e \vec{v}_s \qquad m\frac{\mathrm{d}\vec{v}_s}{\mathrm{d}t} = e\vec{E} \implies \frac{\partial \vec{j}_s}{\partial t} = \frac{n_s e^2}{m}\vec{E}$$

Super current Carrier density velocity Carrier mass

Normal current: $\vec{j}_n = \sigma_n \vec{E}$

Applying curl to both sides.

$$\vec{\nabla} \times \frac{\partial \vec{j}_s}{\partial t} = \frac{n_s e^2}{m}\vec{\nabla} \times \vec{E} \implies \frac{\partial}{\partial t}\left\{\vec{\nabla} \times \vec{j}_s + \frac{n_s e^2 \mu_0}{m}\vec{H}\right\} = 0$$

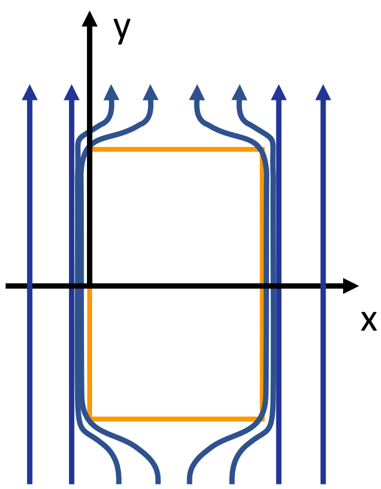
The Meissner effect is independent of history.

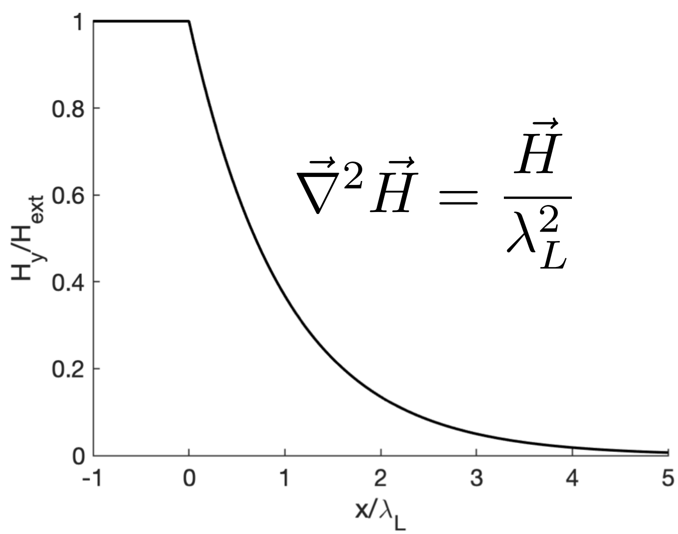
$$\vec{\nabla} \times \vec{j}_s + \frac{n_s e^2 \mu_0}{m}\vec{H} = 0$$

London Penetration Depth

The magnetic field decays into the bulk of the superconductor with a characteristic length scale called the **London Penetration Depth**.

$$\lambda_L \equiv \sqrt{\frac{m}{n_s e^2 \mu_0}}$$



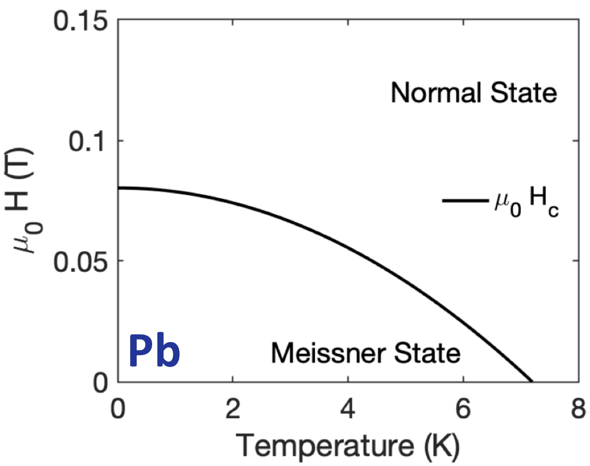


A *super-current* flowing across the surface of the superconductor shields the bulk from the external magnetic field.

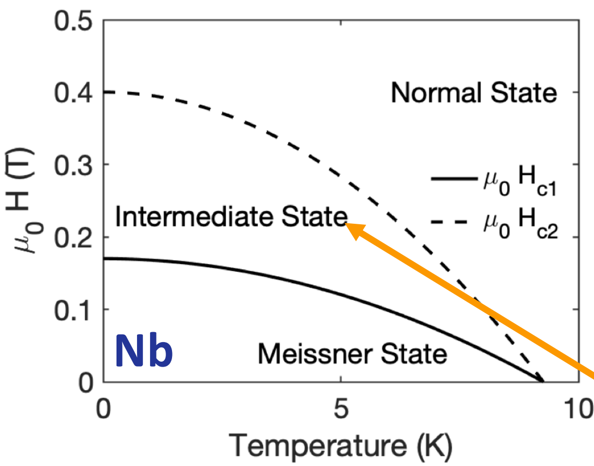
$$\vec{\nabla}^2 \vec{j}_s = \frac{\vec{j}_s}{\lambda_L^2}$$

Types of Superconductors

The Meissner effect has its limits!

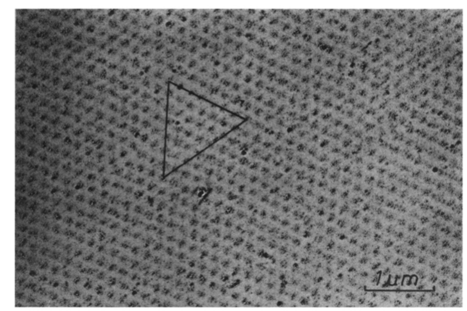


Type - I



Type - II

U. Essmann, Observation of the mixed state, Physica, Vol. 55, 83-93, 1971. https://doi.org/10.1016/0031-8914(71)90244-8.



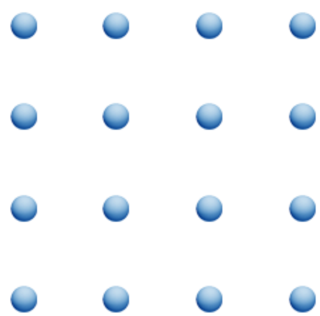
Entry of magnetic vortices.

These **critical fields** impose an upper limit on the peak magnetic field which can be sustained at the cavity surface before the cavity quenches.

BCS Theory

The Bardeen-Cooper-Schrieffer (BCS) theory (1957) describes the microscopic mechanism of low temperature superconductivity.

1. Attractive force between electrons lead to the formation of **Cooper pairs**.



Net attractive force due to lattice deformations (phonons).

Form Cooper pairs with characteristic scales:

a) **Energy Gap (2Δ)**
b) **Coherence Length (ξ)**

2. **Bose – Einstein condensation** below the transition temperature.

BCS Parameters

Category	Parameter Name (0 K)	Nb	Nb3Sn	Description
Energy Scales	Tc (K)	9.25	18.3	Transition temperature
	2Δ (meV)	3	6	Energy gap
Length Scales	λ_L (nm)	39	5.7	London penetration depth
	ξ (nm)	27	65-89	BCS coherence length
	l (nm)	-	-	Mean free path of normal conducting carriers. Depends on purity of material.
Field Scales	$\mu_0 H_{c1}$ (mT)	174	38	Lower critical field
	$\mu_0 H_{c2}$ (T)	0.52	16	Upper critical field
	$\mu_0 H_c$ (mT)	199	520	Thermodynamic critical field
	$\mu_0 H_{sh}$ (mT)	240	440	Superheating field

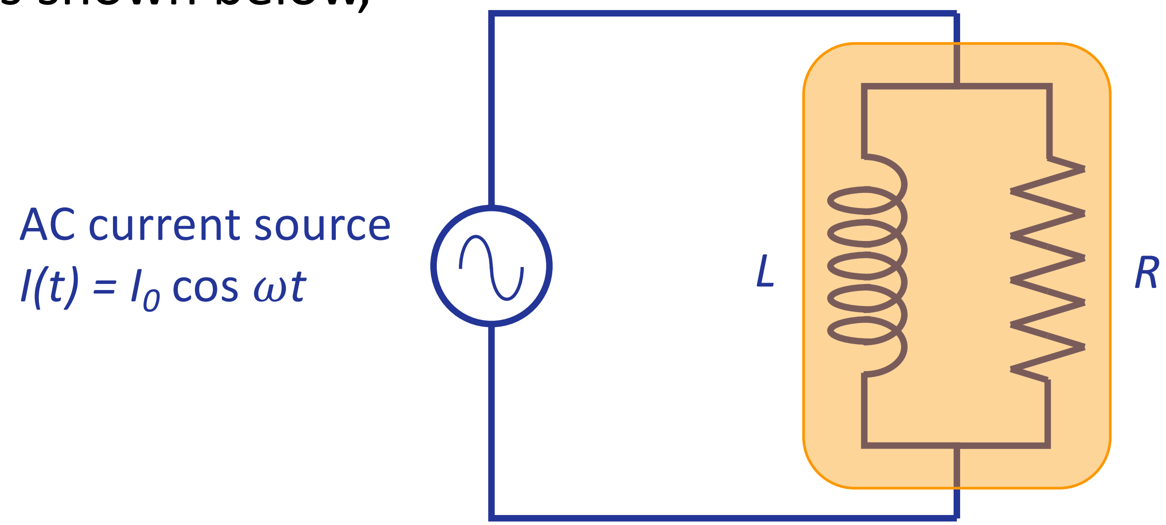
Nb TESLA max gradient: 56 MV/m

S Keckert et al 2019 Supercond. Sci. Technol. 32 075004.
https://iopscience.iop.org/article/10.1088/1361-6668/ab119e

H_{c2} data:
https://accelconf.web.cern.ch/IPAC2015/papers/wepty075.pdf

Exercise II

A simple toy model which describes a superconductor under an alternating field is shown below,



Superconductor modelled as a parallel LR circuit.

When $\omega = 0$, then all the current flows through the inductor and the superconductor exhibits 0 resistance. But what happens when $\omega > 0$? Calculate the power dissipated by the superconductor assuming $R >> \omega L$.

Toy Model: Solution

The potential difference (in phasor notation) across the resistor is ($R \gg \omega L$),

$$\tilde{V} = \frac{I_0}{\frac{1}{i\omega L} + \frac{1}{R}} = \frac{iI_0 R\omega L}{R + \cancel{i\omega L}} \approx iI_0\omega L$$

When $\omega = 0$, the potential difference across the resistor is 0 so no current flows through it and there is no dissipation. But when $\omega > 0$, the power dissipated is,

$$P = \boxed{\frac{1}{2}}\frac{|\tilde{V}|^2}{R} = \frac{I_0^2 L^2 \boxed{\omega^2}}{2\boxed{R}}$$

Averaging the sine wave

Dissipation is:

1. Proportional to square of frequency.
2. Proportional to conductivity of normal component.

Two Fluid Model and BCS Resistance

The toy model was a simple interpretation of the classical two-fluid picture (Gorter and Casimir 1934), where the current is transported via

1. Superconducting carriers – Actually Cooper pairs,
2. Normal conducting carriers – Quasi-particle excitations of the BCS ground state.

The surface resistance of a superconductor (Mattis and Bardeen 1958) is (*low field, low temperature, low frequency, dirty limit*),

$$R_s = \boxed{R_{\text{BCS}}} + \boxed{R_{\text{res}}},$$

Residual resistance due to material imperfections or trapped flux.

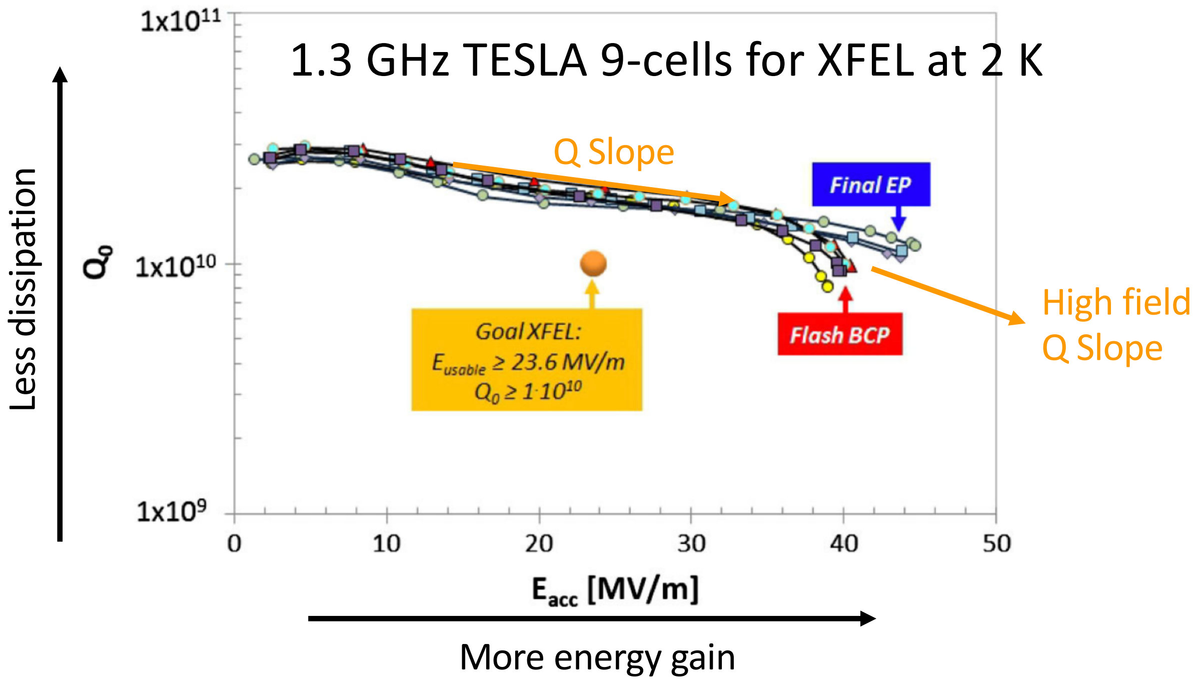
$$R_{\text{BCS}} = \frac{\mu_0^2 \lambda^3 \sigma_n \omega^2 \Delta}{k_B T} \ln\left[\frac{C_1 k_B T}{\hbar\omega}\right] \exp\left[-\frac{\Delta}{k_B T}\right]$$

$$C_1 \sim 9/4,\ T < T_c/2,\ \omega < \Delta/\hbar$$

http://accelconf.web.cern.ch/srf2019/talks/thtu1_talk.pdf

Q vs E

Surface resistance is a function of applied field! – Superconductors are non-linear.

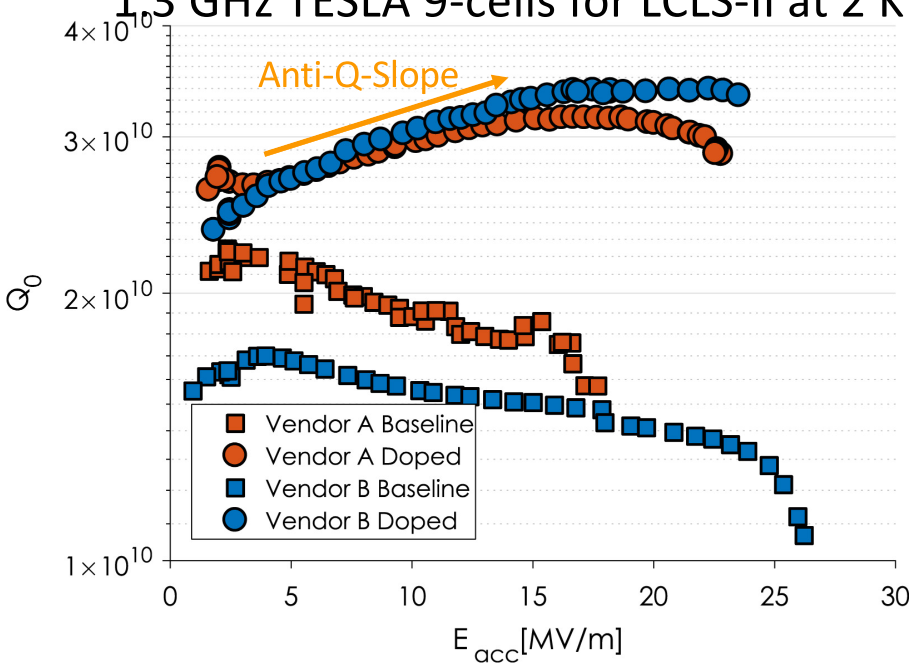


W. Singer et al., Phys. Rev. Accel. Beams 19, 092001 (2016), https://doi.org/10.1103/PhysRevAccelBeams.19.092001

Current Frontiers: Nitrogen Doping

Doping with nitrogen gives rise to *anti-Q-slope* behavior.

1.3 GHz TESLA 9-cells for LCLS-II at 2 K

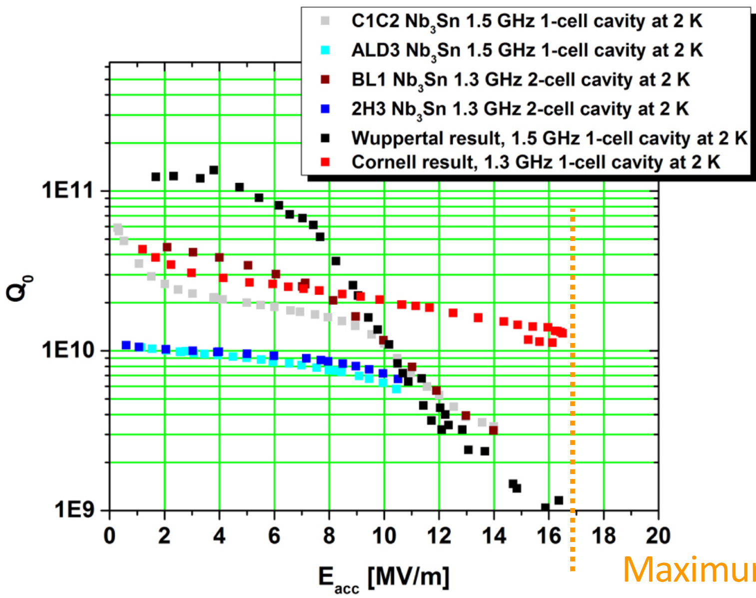


D. Gonnella et al., Industrialization of the nitrogen-doping preparation for SRF cavities for LCLS-II, NIM A, Vol 883, 143-150, 2018. https://doi.org/10.1016/j.nima.2017.11.047.

New recipes and dopants are being actively researched!

Current Frontiers: Nb_3Sn

Niobium-Tin exhibits lesser surface resistance than Nb at the same temperature and is theoretically capable of reaching higher fields.



Maximum field much lower than Nb!

Single cell tests at 2 K

S Posen and D L Hall 2017 Supercond. Sci. Technol. 30 033004 https://iopscience.iop.org/article/10.1088/1361-6668/30/3/033004

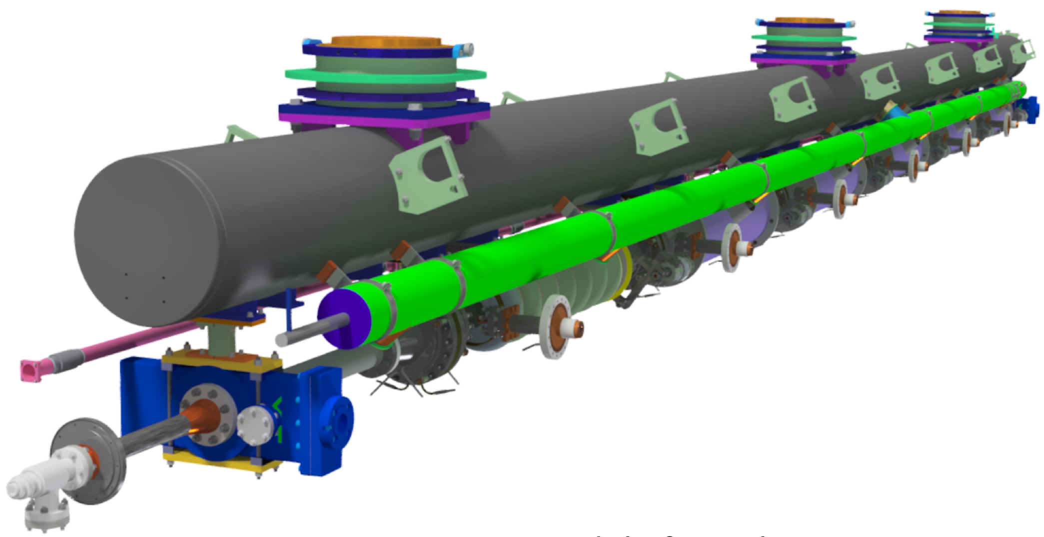
Cavities made of Nb3Sn quench at fields much lower than the theoretical maximum. Active field of research!

Using Cavities

Lots of other challenges!

(https://sites.google.com/view/srfuspas17/home)

- Field Emission and Multipacting – This is why SRF cavities are elliptical!
- Tuning and Detuning
- Power (https://uspas.fnal.gov/programs/2019/newmexico/courses/microwave-sources.shtml) and Control (https://uspas.fnal.gov/programs/2018/odu/courses/control-theory.shtml)
- Higher Order Modes and Wakefields (https://uspas.fnal.gov/materials/19Knoxville/Knoxville-Wakefields.shtml)
- Cryogenic System (https://uspas.fnal.gov/materials/19NewMexico/NewMexico-Cryogenic.shtml)

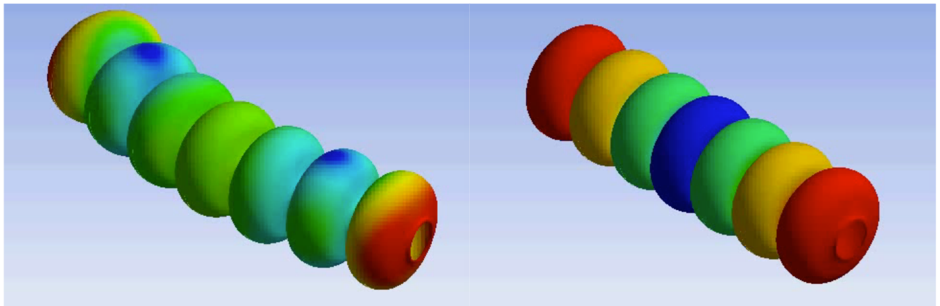


Main Linac Cryomodule from the CBETA project.
A. Bartnik et al., Phys. Rev. Lett. 125, 044803, 2020 https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.125.044803

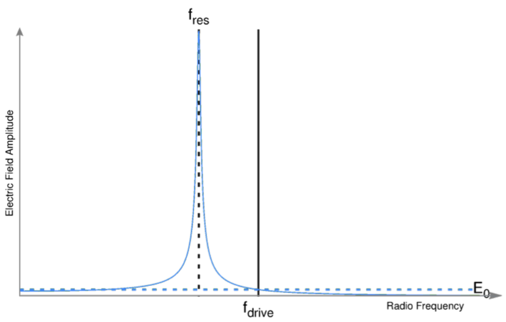
Resonance Detuning

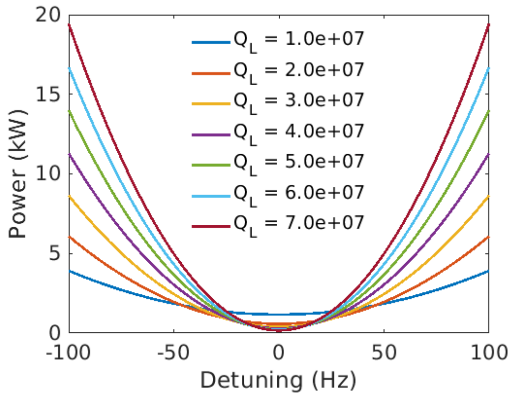
Vibrations can dynamically deform SRF cavities.

7-cell SRF cavities used in the Main Linac Cryomodule from the CBETA project.



Resonance frequency moves around!





Vibrations increase rf power requirements in narrow bandwidth linacs.

Things to Remember

- V, R/Q and Q are important parameters which determine cavity operation.
- Two-fluid picture: SRF surface resistance arises from normal conducting contribution. Depends on normal conductivity, temperature, transition temperature and frequency.
- Superconductors are non-linear so the intrinsic quality factor changes with field.
- Big leap from cavity design and materials to actual use in accelerators.

References

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