USPAS Accelerator Physics 2024
Hampton VA / Northern Illinois University

Lattice Examples II
(or putting much of the week together)
(or Even More Stupid Lattice Tricks)

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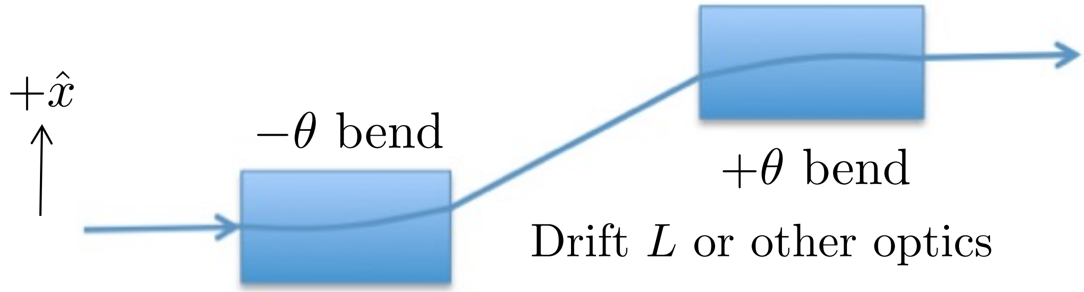
Happy Birthday to Polykarp Kusch (Nobel 1955; EDM), Paul Newman, and Wayne Gretzky!
Happy National Peanut Brittle Day, National Spouses Day, and Dental Drill Appreciation Day!

Overview (Afternoon)

- Achromats
  - Doglegs and achromatic doglegs
  - Chicanes and bunch compressors
  - Double bend achromat
  - Triple bend achromat
  - (MAX-IV Multi-bend achromat; more Tuesday)

- 2D/3D manipulation:
  - Longitudinal/transverse emittance exchange

Doglegs



- Displaces beam transversely without changing direction
- What is effect on 6D optics?

$$\mathbf{M}_{\text{dipole}}(\rho,\theta) = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\theta & -\rho(1-\cos\theta) & 0 & 0 & 1 & \boxed{L/(\gamma^2\beta^2)} - \rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Be careful about the coordinate system and signs!!
- If ρ,θ>0, positive displacement points **out** from dipole curvature
- Be careful about order of matrix multiplication!

Reverse Bend Dipole Transport

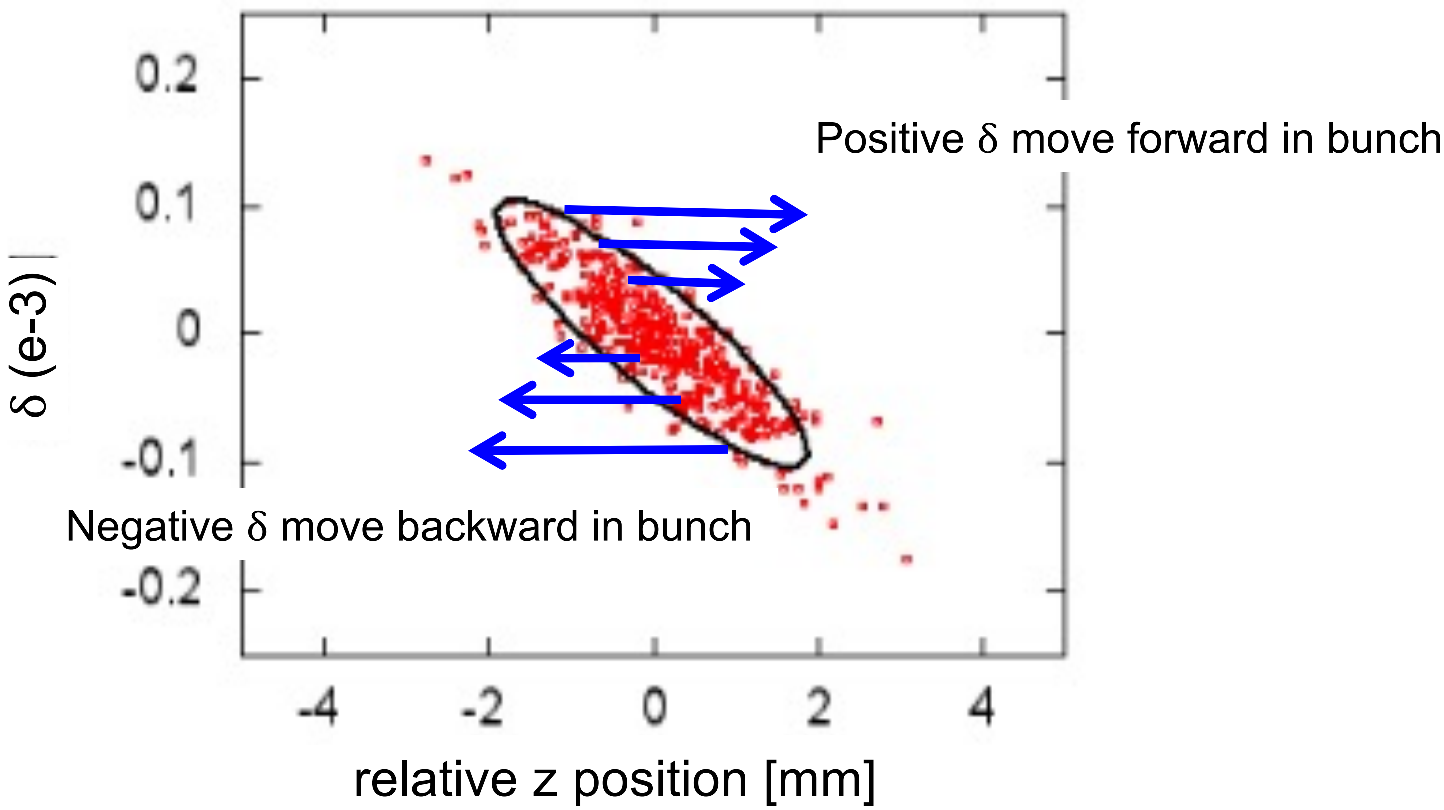
- What is the correct 6x6 transport matrix of a reverse bend dipole?
- It turns out to be achieved by reversing **both** ρ **and** θ
  - ρθ=L (which stays positive) so both must change sign

$$\mathbf{M}_{\mathrm{dipole}}(-\rho,-\theta) = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \boxed{-\rho(1-\cos\theta)} \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & \boxed{-\sin\theta} \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \boxed{\sin\theta} & \boxed{\rho(1-\cos\theta)} & 0 & 0 & 1 & \boxed{L/(\gamma^2\beta^2)} - \rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

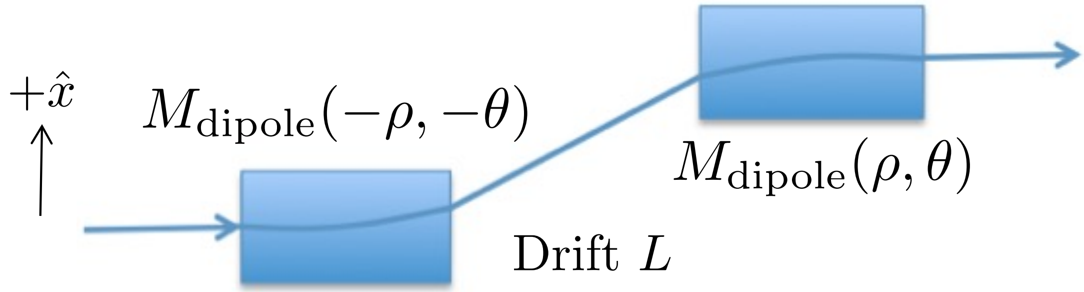
$$M_{\mathrm{drift}} = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \boxed{L/(\gamma^2\beta^2)} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Aside: Longitudinal Phase Space Drift

- Wait, what was that M_{56} term with the relativistic effects?
  - Recall longitudinal coordinates are (z, δ)
  - This extra term is called "ballistic drift": not in all codes!
    - Important at low to modest energies and for bunch compression
    - Relativistic terms enter converting momentum p to velocity v



Weak Dogleg



$$\mathbf{M}_{\text{dogleg}} = \mathbf{M}_{\text{dipole}}(\rho, \theta)\mathbf{M}_{\text{drift}}\mathbf{M}_{\text{dipole}}(-\rho, -\theta)$$

Check sign

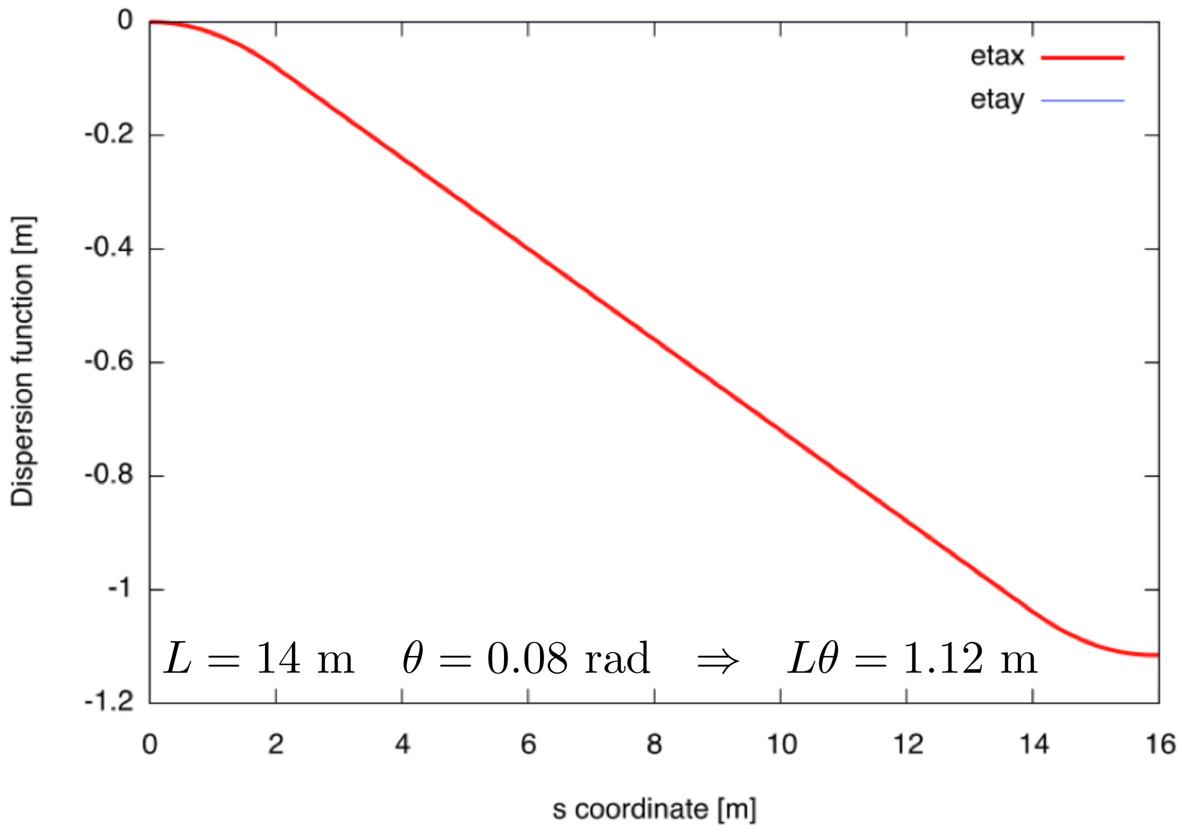
$$\mathbf{M}_{\text{weak dogleg}} = \begin{pmatrix} 1 & \rho\theta & \frac{\rho\theta^2}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \rho\theta & \frac{\rho\theta^2}{2} \\ 0 & 1 & -\theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & L + 2\rho\theta & -L\theta \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{matrix} (\eta, \eta')_{\text{in}} = 0 \\ \Rightarrow (\eta, \eta')_{\text{out}} = (-L\theta, 0) \end{matrix}$$

Strong dogleg can also be derived:

$$D = -L\cos\theta\sin\theta \qquad D' = \frac{L\sin^2\theta}{\rho}$$

Dogleg Dispersion



For weak dogleg

$$(\eta, \eta')_{\text{in}} = 0$$
$$\Rightarrow \quad (\eta, \eta')_{\text{out}} = (-L\theta, 0)$$

Does this make sense?

$$\Delta x'(\delta) = \frac{BL}{(B\rho)} = \frac{q}{p(1+\delta)}[BL] \approx \frac{q}{p}[BL](1-\delta) = (1-\delta)\Delta x'(\delta = 0)$$

A small momentum offset of $+\delta$ reduces the dipole kick by a factor of delta, and this is magnified to a transverse offset from design at the end of the dogleg by $-\delta L\theta$.

Achromatic Dogleg

- How can we make an achromatic dogleg?

$$(\eta, \eta')_{\text{in}} = (0\text{ m}, 0) \quad \Rightarrow \quad (\eta, \eta')_{\text{out}} = (0\text{ m}, 0)$$

- Use an I insertion (e.g. four consecutive π/2 insertions)

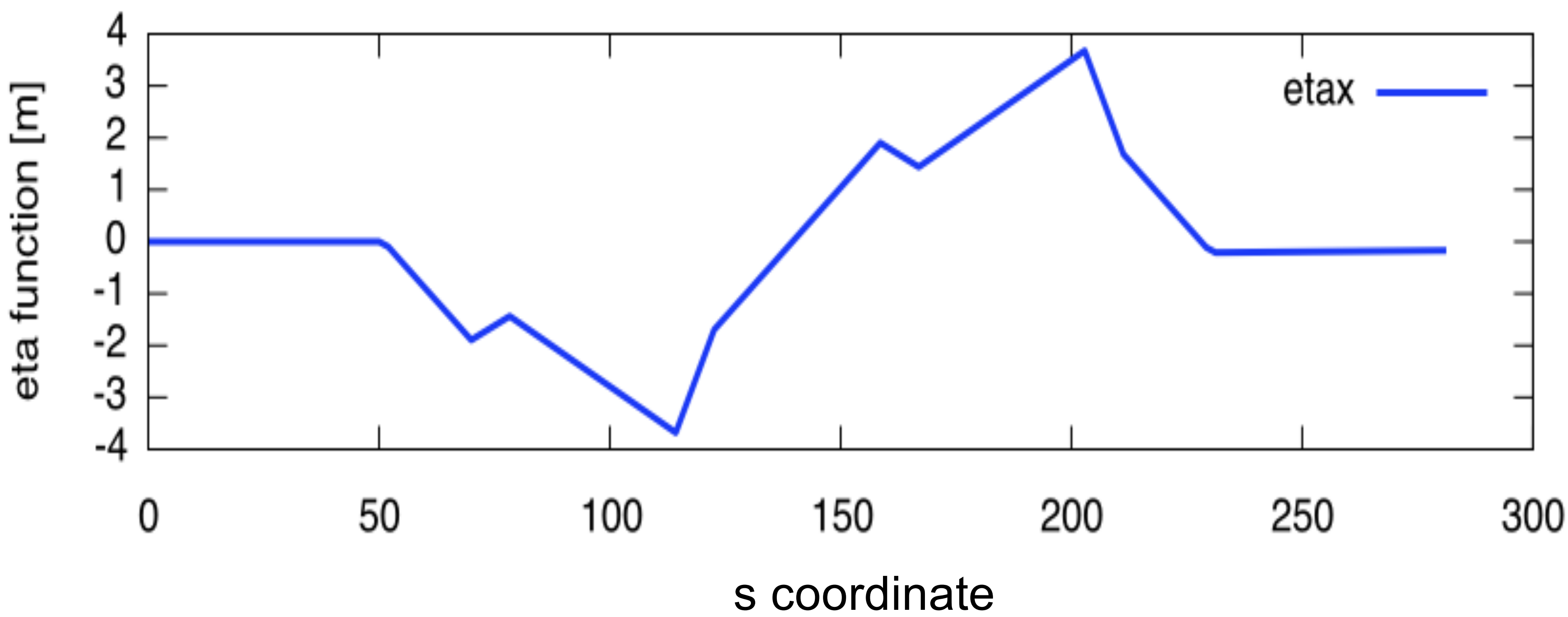
$$\mathbf{M}_{\pi/2} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} = \mathbf{J} \qquad (\text{Recall } \mathbf{J}^4 = \mathbf{I})$$

$$\mathbf{M}_{\text{achromatic dogleg}} = \begin{pmatrix} \cos(2\theta) & \rho \sin(2\theta) & 0 \\ -\frac{\sin(2\theta)}{\rho} & \cos(2\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

achromatic!

- **Any** transport with net phase advance of 2nπ will be achromatic (nπ if all dipoles bend in same direction)
  - common trick for matching dispersive bending arcs to non-dispersive straight sections.

Achromatic Dogleg



Achromatic Dogleg: Steffen CERN School Notes

Example of nondispersive translating system

Φ = sector magnet bend. angle

$\varphi = \ell\sqrt{k}$ = quadrupole magnet phase angle

d, λ = drift space lengths.

The system is nondispersive if the sinelike trajectory (with respect to the central symmetry point) goes through the mid-point of the bending magnets, i.e. if

$$\rho \tan\frac{\Phi}{2} + \lambda = \frac{1}{\sqrt{k}}\,\frac{d\sqrt{k}\cos\varphi + 2\sin\varphi}{d\sqrt{k}\sin\varphi - 2\cos\varphi}\,.$$

Focusing also in the other plane may be obtained by adding a third quadrupole of opposite polarity at the symmetry point.

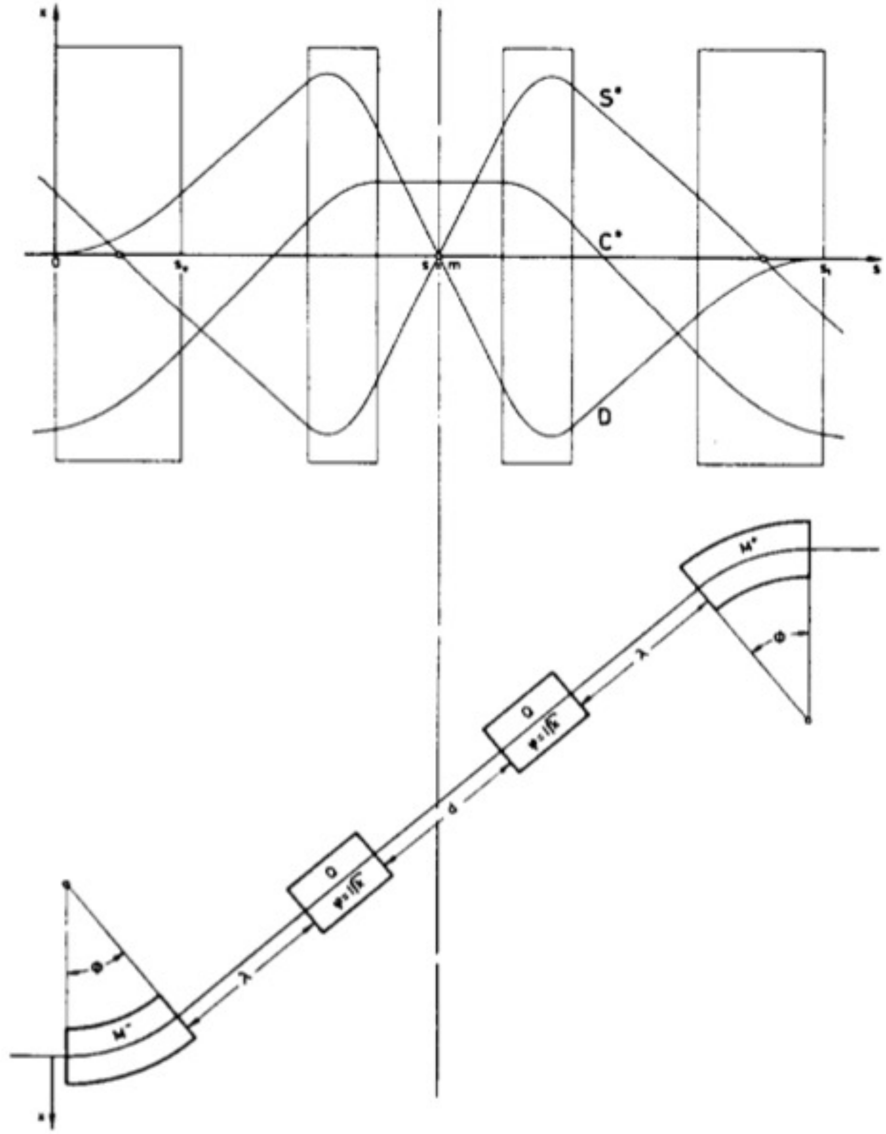


Fig. 15: Nondispersive translating system.

K. Steffen, CERN-85-19-V-1, 1985, p. 55

First-Order Achromat Theorem

- A lattice of n repetitive cells is achromatic (to first order, or in the linear approximation) iff $\mathbf{M}^n = I$ or each cell is achromatic
- Proof:

$$\text{Consider } \mathbf{R} \equiv \begin{pmatrix} \mathbf{M} & \bar{d} \\ 0 & 1 \end{pmatrix} \quad \text{where } \begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_2 = \mathbf{R} \begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_1 \quad \bar{d} = \begin{pmatrix} M_{16} \\ M_{26} \end{pmatrix}$$

$$\textbf{For } n \text{ cells : } \quad \mathbf{R}^n = \begin{pmatrix} \mathbf{M}^n & (\mathbf{M}^{n-1} + \mathbf{M}^{n-2} + \ldots + \mathbf{I})\bar{d} \\ 0 & 1 \end{pmatrix}$$

$$\text{but } \; (\mathbf{M}^{n-1} + \mathbf{M}^{n-2} + \ldots + \mathbf{I}) = (\mathbf{M}^n - I)(\mathbf{M} - I)^{-1}$$

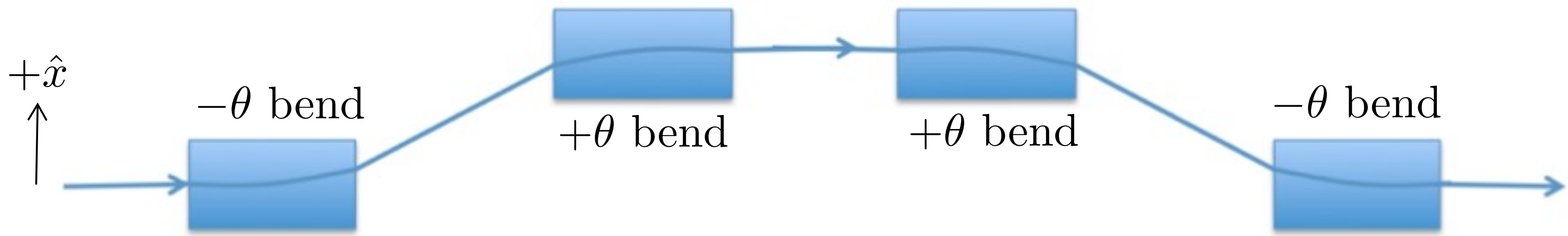
$$\text{So for } n \text{ cells : } \quad \mathbf{R}^n = \begin{pmatrix} \mathbf{M}^n & (\mathbf{M}^n - \mathbf{I})(\mathbf{M} - \mathbf{I})^{-1}\bar{d} \\ 0 & 1 \end{pmatrix}$$

- So the lattice is achromatic only if $\bar{d} = 0$ or $\mathbf{M}^n = \mathbf{I}$

$$\mathbf{M}^n = \mathbf{I} \cos \mu_{\text{tot}} + \mathbf{J} \sin \mu_{\text{tot}} \quad \Rightarrow \quad \mu_{\text{tot}} = 2\pi k$$

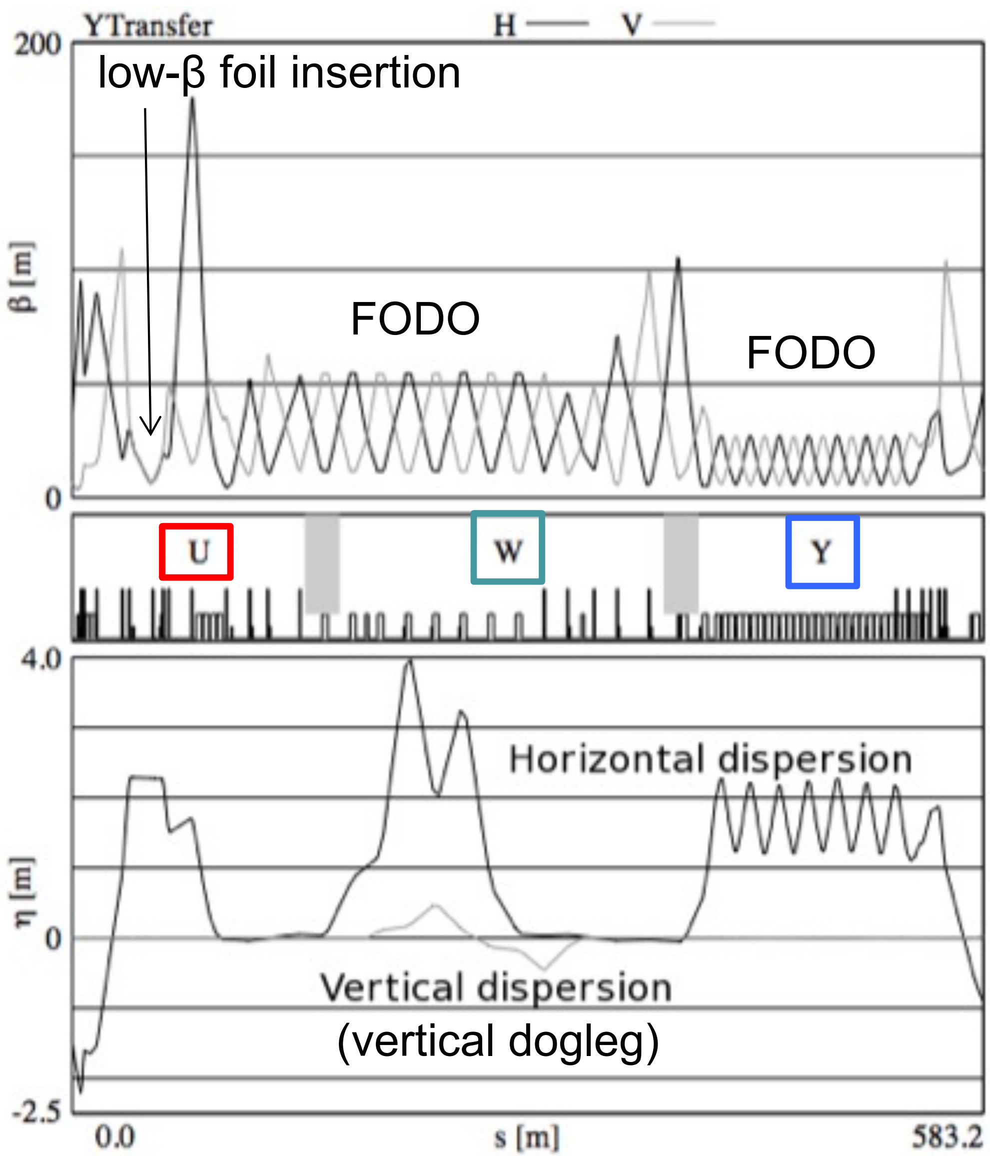
S.Y. Lee, “Accelerator Physics”

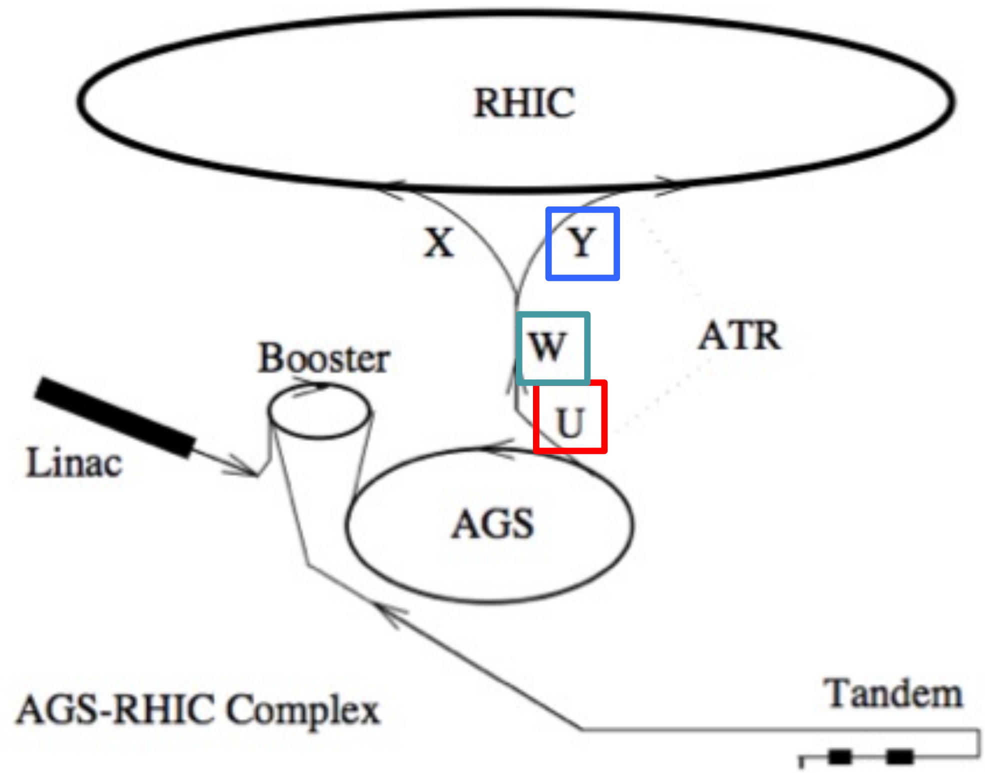
Chicane



- Divert beam around an obstruction
  - e.g. vertical bypass chicane in Fermilab Main Ring
  - e.g. horizontal injection chicane in CEBAF recirculating linac
  - Essentially a design orbit "4-bump" (4 dipoles)
- Usually need some focusing, optics between dipoles
- Usually design optics to be achromatic
  - Operationally null orbit motion at end of chicane vs changes in input beam energy
- Naively expect $M_{56}<0$ (bunch lengthening or decompression)
  - Higher energy particles ($+\delta$) have shorter path lengths
  - But can compress bunches with introduction of longitudinal correlation

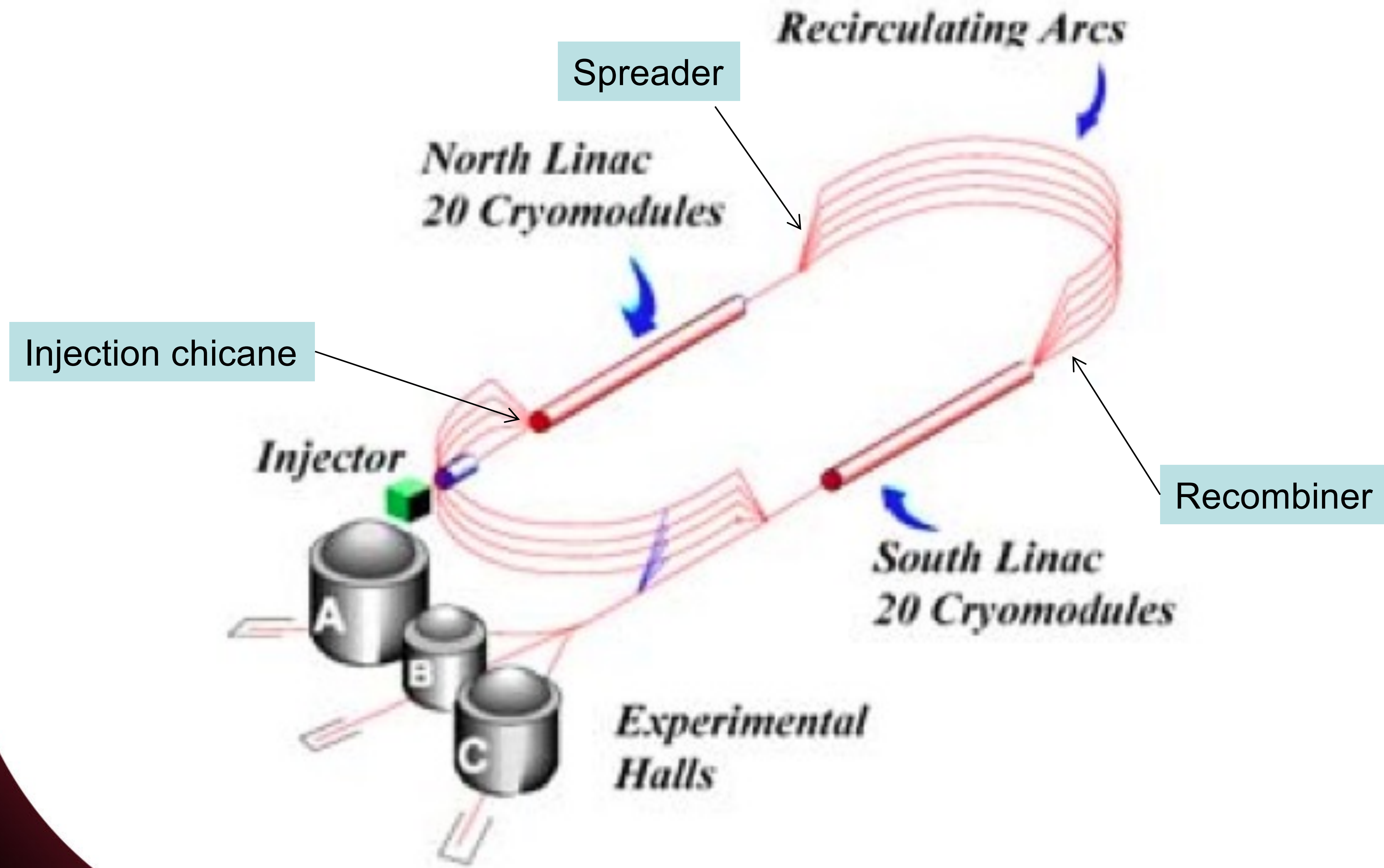
AGS to RHIC (ATR) Transfer Line





The ATR vertical dogleg is not strictly a dogleg since the planes of the AGS and RHIC accelerators are not parallel

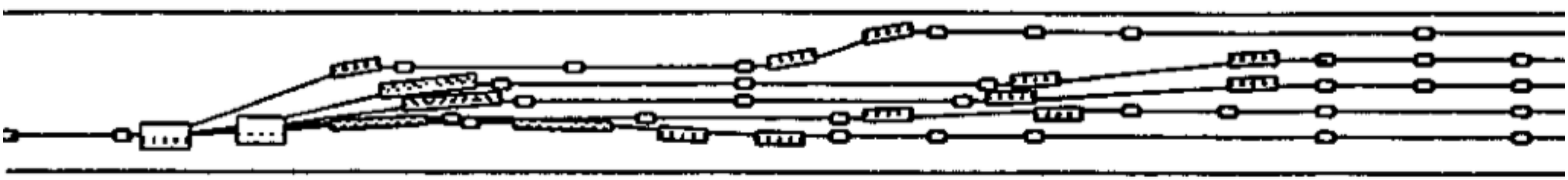
CEBAF



Recirculating Arcs
Spreader
North Linac
20 Cryomodules
Injection chicane
Injector
Recombiner
South Linac
20 Cryomodules
A
B
C
Experimental
Halls

CEBAF Spreaders/Recombiners

- Problem: Separate different energy beams for transport into arcs of CEBAF, and recombine before next linac
  - Achromats: arcs are FODO-like, linacs are dispersion-free
  - “I” insertion: 1 betatron wavelength between dipoles
  - Single dogleg: unacceptably high beta functions
  - Two consecutive “staircase” doglegs with same total phase advance was solution

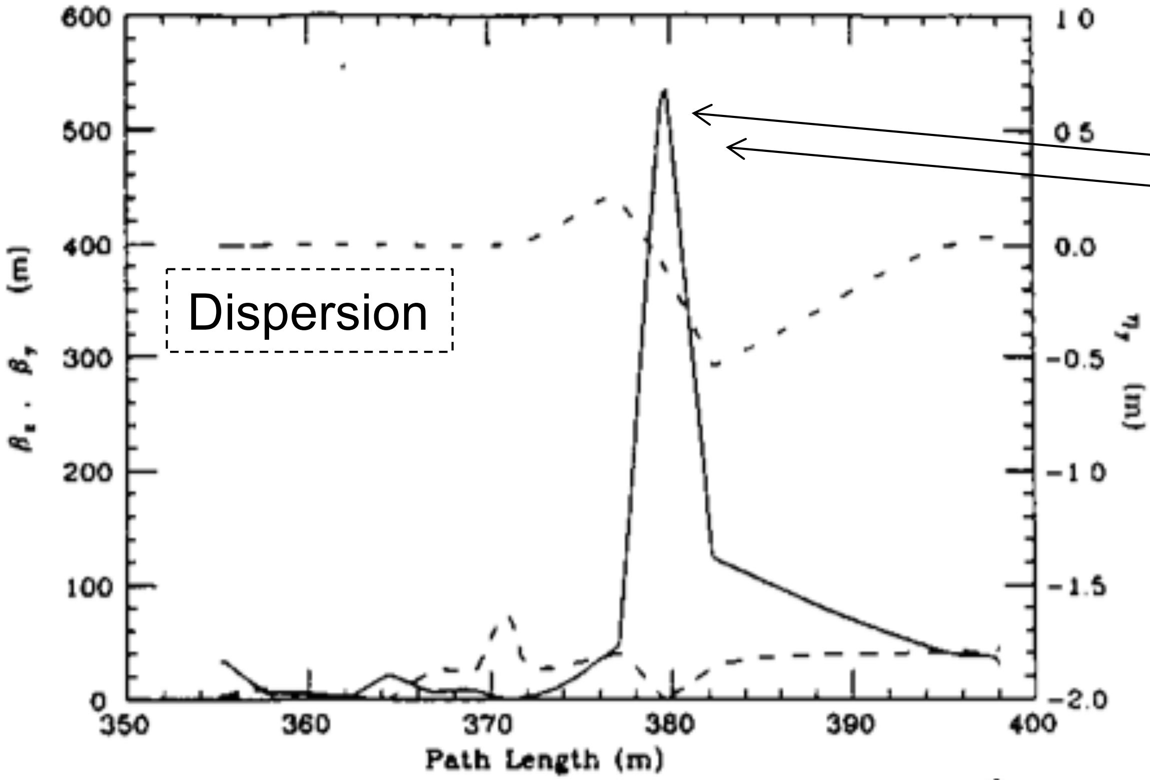


EAST ARC ELEVATION

  - Still quite a challenge in physical layout of real magnets!

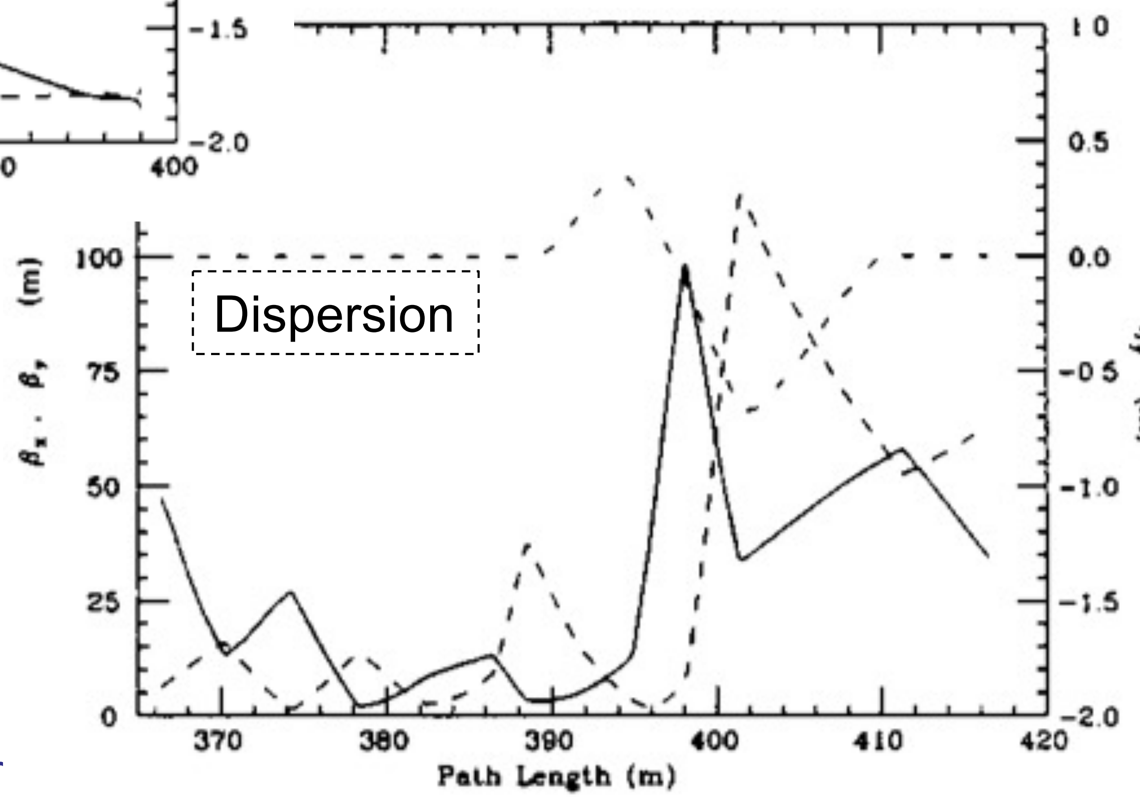
D. Douglas, R.C. York, J. Kewisch, “Optical Design of the CEBAF Beam Transport System”, 1989

CEBAF Spreaders/Recombiners



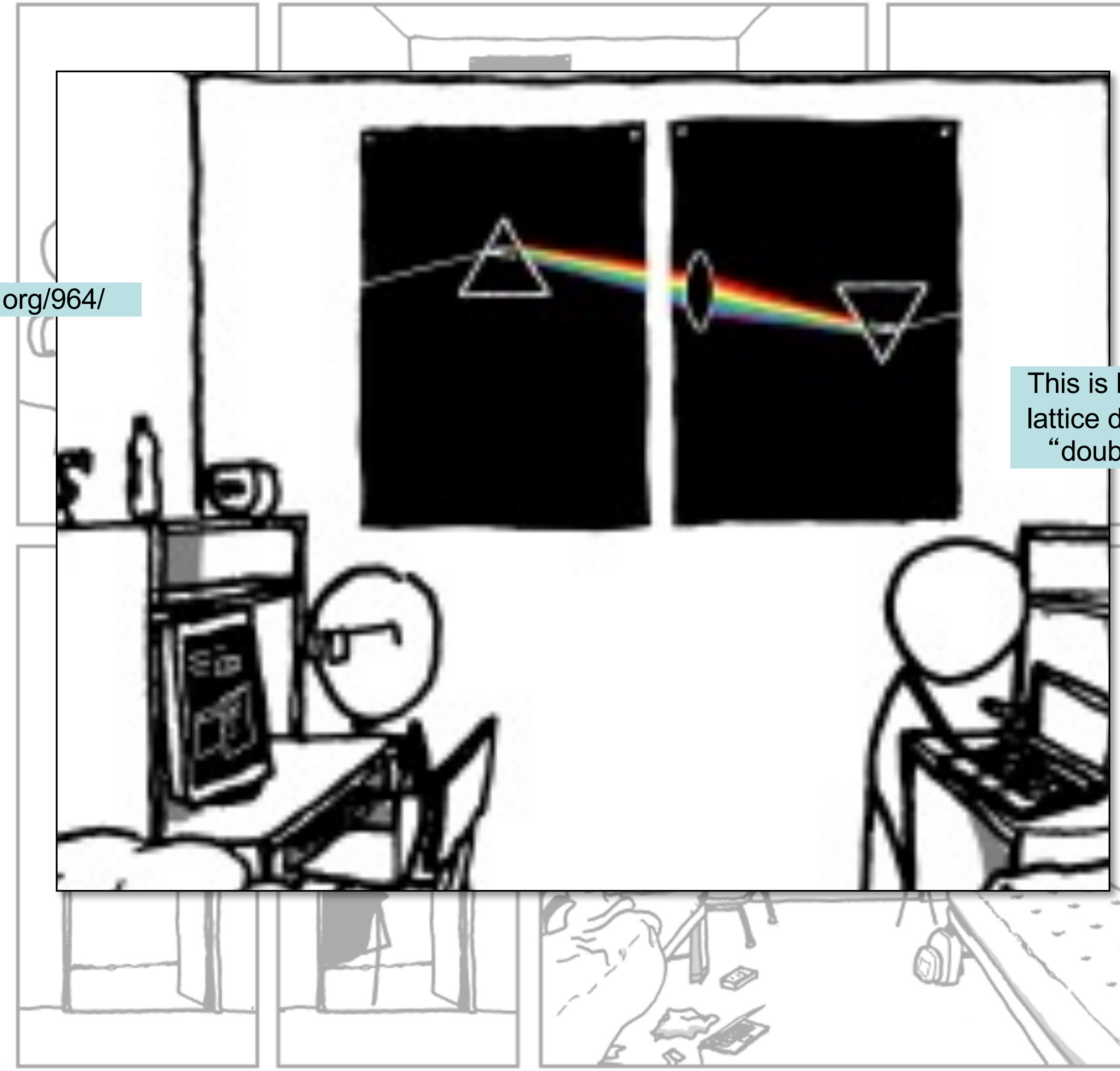
"One step" recombiner
Unacceptably large vertical beta function/beam size (550 m)

"Staircase" two-step recombiner
Acceptable beta functions and beam sizes in both planes (100 m)



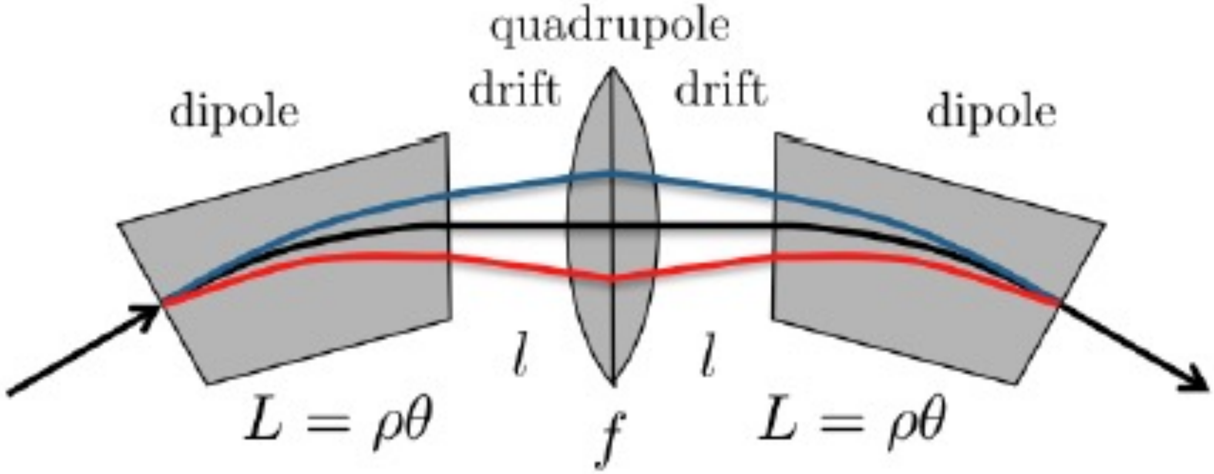
(xkcd yet again: double bend achromat)

http://www.xkcd.org/964/



This is known in accelerator lattice design language as a "double bend achromat"

Double Bend Achromat (approximate)



- Let's calculate constraints for the double bend achromat

$$M_{\mathbf{dipole}} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho[1-\cos\theta] \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

Keep lowest-order terms in θ, including θ^2 in upper right term since $\rho\theta$=L

$$M_{\mathbf{dipole}} = \begin{pmatrix} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

Double Bend Achromat (approximate)

$$\mathbf{M}_{\mathrm{DBA}} = \begin{pmatrix} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{M}_{\mathrm{DBA}} = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} C &= 1 - \frac{(L+l)}{f} \\ S &= \frac{(L+l)(2f-L-l)}{f} \\ D &= \theta\frac{(L+l)(4f-L-2l)}{2f} \\ C' &= -\frac{1}{f} \\ S' &= 1 - \frac{(L+l)}{f} = C \\ D' &= \theta\frac{(4f-L-2l)}{2f} = \frac{D}{L+l} \end{aligned}$$

Double Bend Achromat (approximate)

- The periodic solutions for dispersion for the general M matrix were shown in class earlier today

$$\eta(\text{periodic}) = \frac{[1-S']D + SD'}{2(1-\cos\mu)} = 0$$

$$\eta'(\text{periodic}) = \frac{[1-C]D' + C'D}{2(1-\cos\mu)} = 0$$

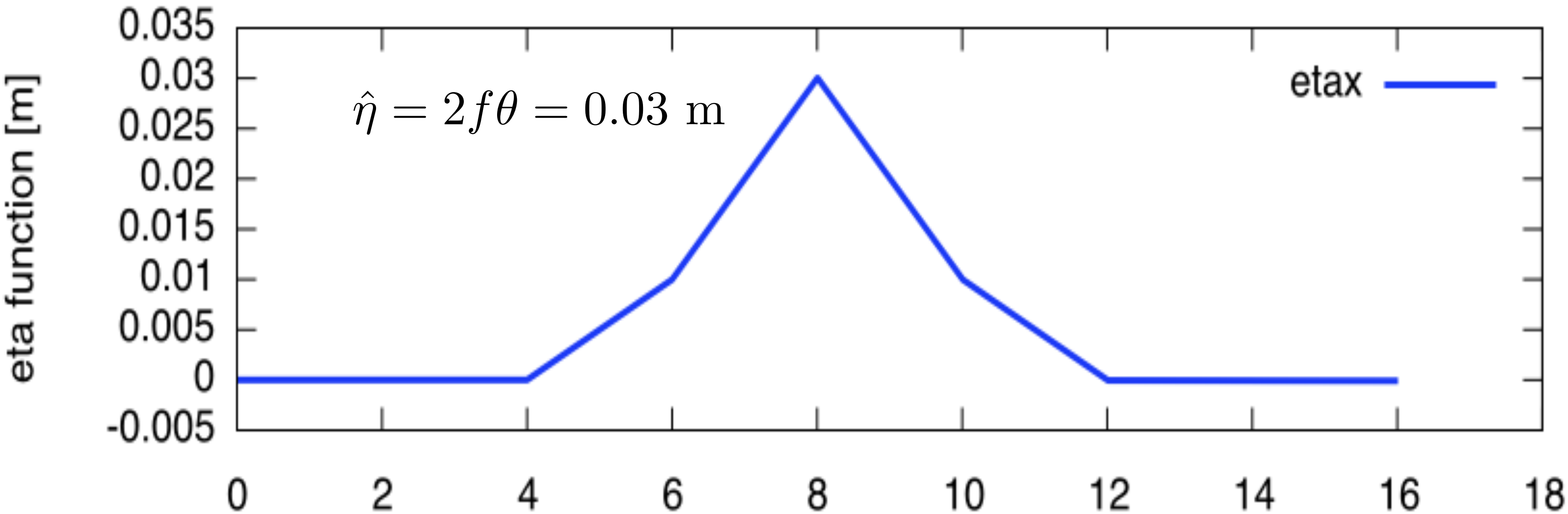
- It turns out that the η' equation is satisfied automatically!
  - This is a consequence of the mirror symmetry of the system
- The η equation is satisfied if D=0:

$$D = \theta\frac{(L+l)(4f-L-2l)}{2f} = 0$$

$$\Rightarrow 4f - L - 2l = 0 \quad \Rightarrow \quad f = \frac{L+2l}{4} \qquad \hat{\eta} = \frac{(L+2l)\theta}{2} = 2f\theta$$

Include link to madx file

Double Bend Achromat



$$L = l = 2 \text{ m} \quad \theta = 0.01 \text{ rad} \quad f = \frac{L+2l}{4} = 1.5 \text{ m} \quad (KL_{\text{quad}}) = 0.667 \text{ m}^{-1}$$

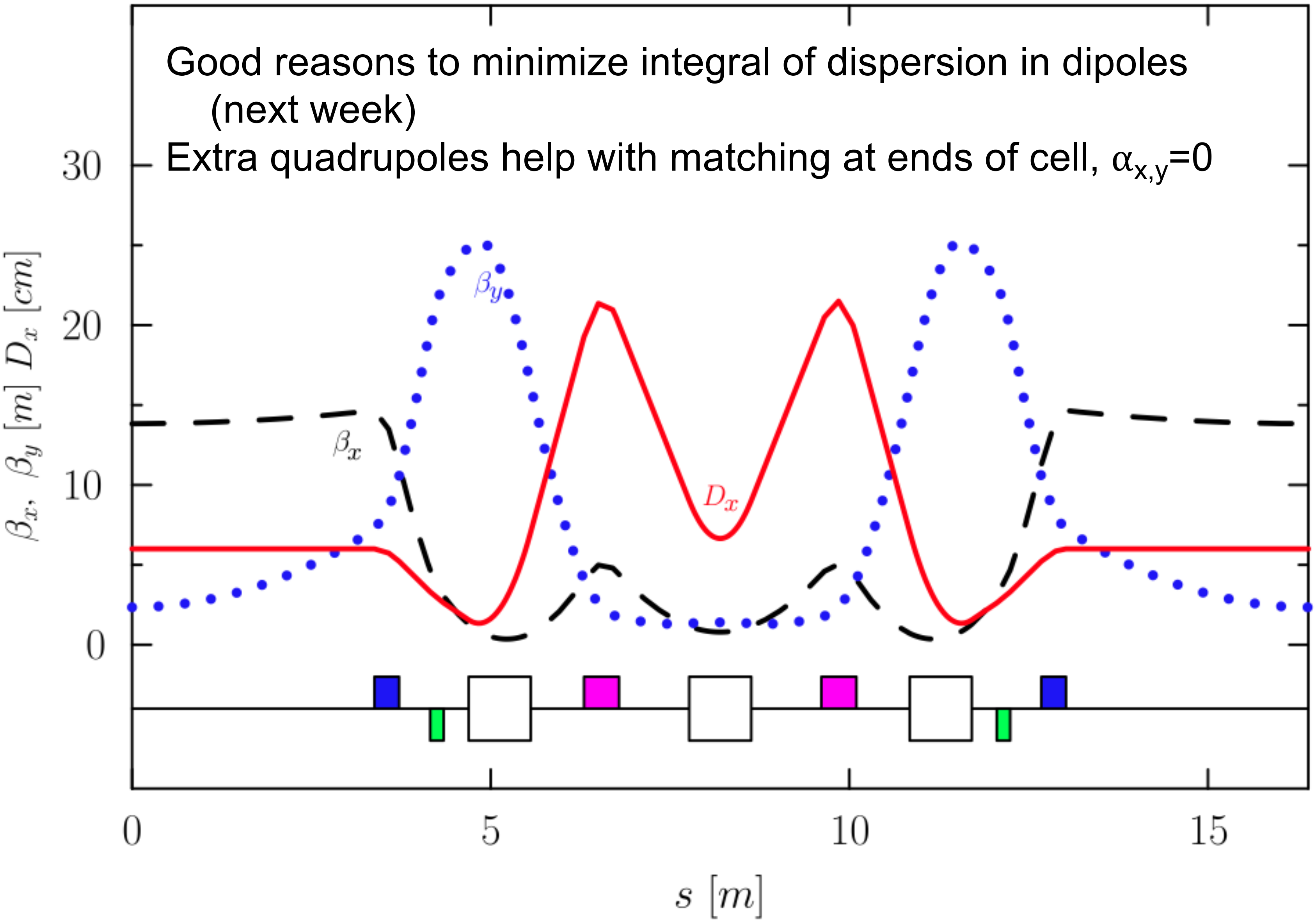
$$\text{Exact DBA}: \quad f = \frac{l}{2} + \frac{\rho}{2}\tan(\theta/2) \qquad \hat{\eta} = \rho(1-\cos\theta) + l\sin\theta$$

- DBA is also known as a Chasman-Green lattice
  - Used in early third-generation light sources (e.g. NSLS at BNL)
  - More after we discuss synchrotron radiation, $\mathcal{H}$ functions

Triple Bend Achromat Cell (ALS at LBL)

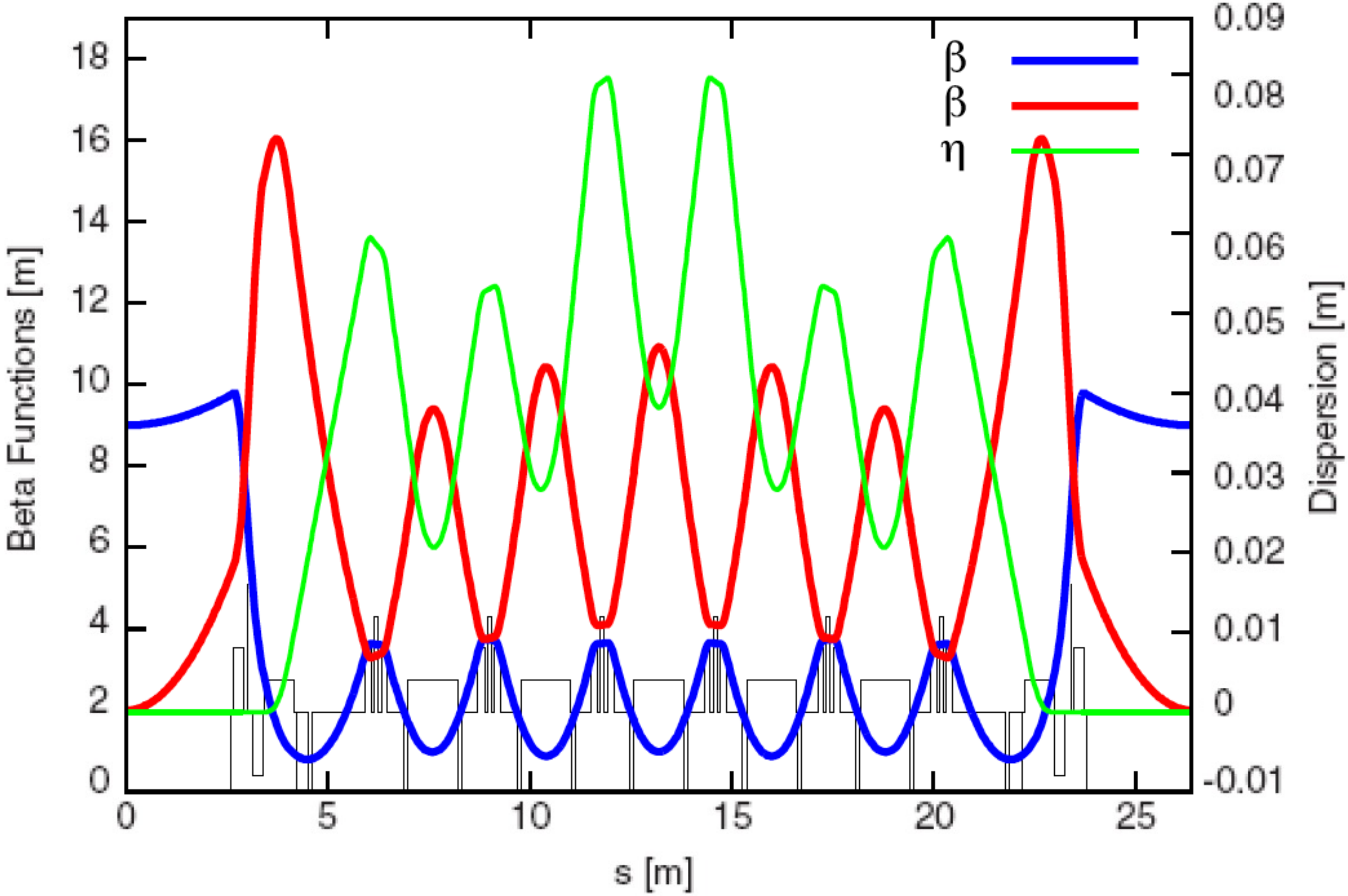
Good reasons to minimize integral of dispersion in dipoles (next week)

Extra quadrupoles help with matching at ends of cell, $\alpha_{x,y}=0$



L. Yang et al, Global Optimization of an Accelerator Lattice Using Multiobjective Genetic Algorithms, 2009

MAX-IV 7-Bend Achromat

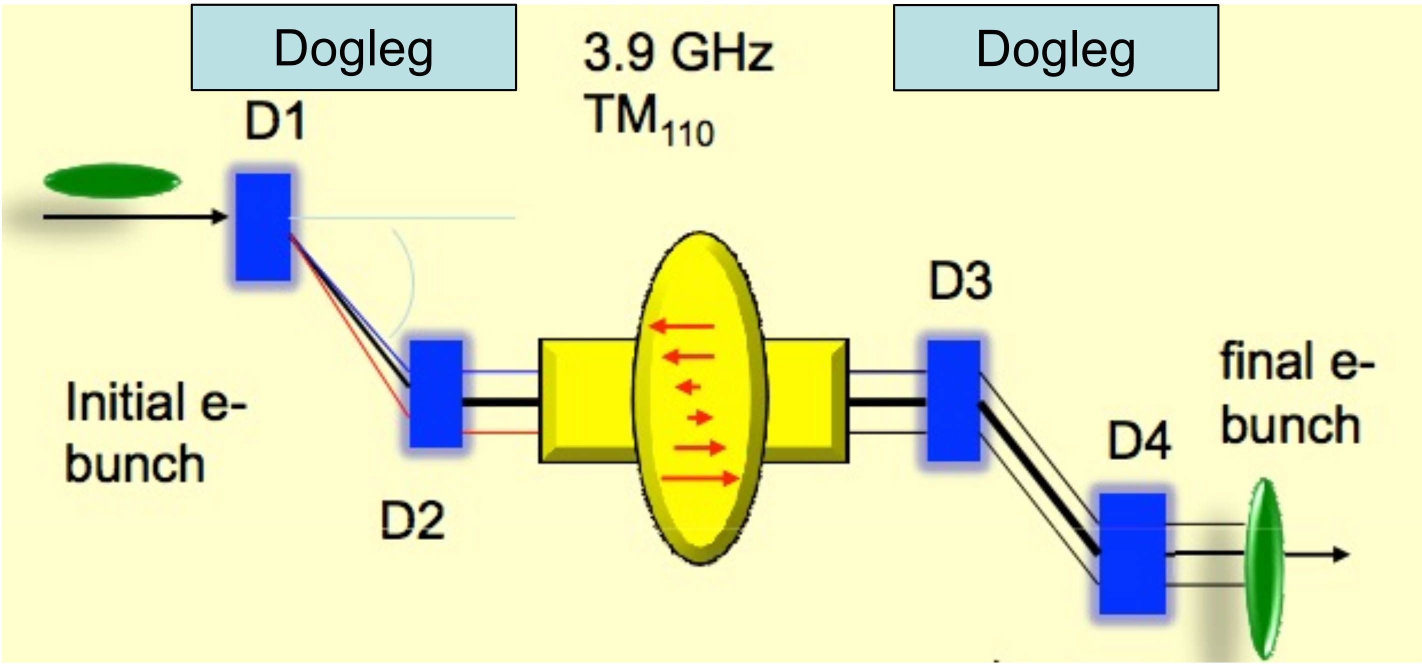


Transverse/Longitudinal Emittance Exchange

- X-ray FELs demand ultra-low transverse emittance beam*
- State-of-the art photo-injectors can generate low 6-D emittance. Typically asymmetric emittances. Emittance exchange can swap transverse with the longitudinal emittance.
- Allows one to convert transverse modulations to longitudinal modulations : Beam shaping application
- Can also be used to suppress microbunching instability**

J.C.T. Thangaraj, Experimental Studies on an Emittance Exchange Beamline at the A0 Photoinjector, 2012

Fermilab A0 Emittance Exchanger



θ : Bending angle

η : dogleg dispersion

L : dogleg length

L_c : RF cell length

$$\mathbf{M} = \begin{pmatrix} 1 & \frac{L_c}{4} & -\frac{(4L+L_c)}{4\eta} & \eta - \frac{\theta(4L+L_c)}{4} \\ 0 & 1 & -\frac{1}{\eta} & -\theta \\ -\theta & \eta - \frac{\theta(4L+L_c)}{4} & 1 + \frac{\theta L_c}{4\eta} & \frac{\theta^2 L_c}{4} \\ -\frac{1}{\eta} & -\frac{4L+L_c}{4\eta} & \frac{\theta L_c}{4\eta^2} & 1 + \frac{\theta L_c}{4\eta} \end{pmatrix} \begin{pmatrix} x \\ x' \\ z \\ \delta \end{pmatrix}$$

J.C.T. Thangaraj, Experimental Studies on an Emittance Exchange Beamline at the A0 Photoinjector, 2012

TM110 RF Cavity Mode

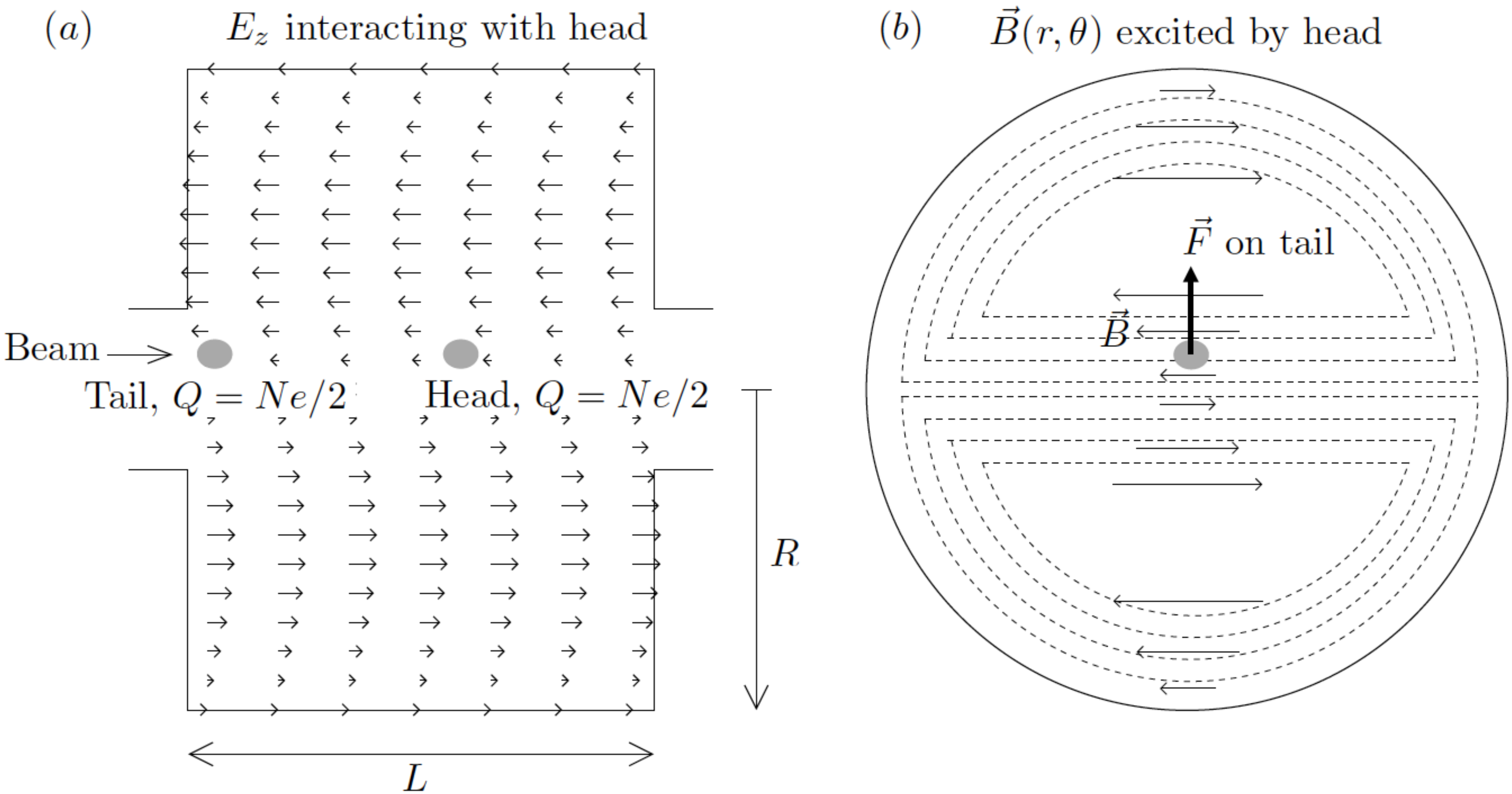


Figure 14.6 in textbook

Chicane Style Emittance Exchange (2011)

DAO XIANG *et al.* Phys. Rev. ST Accel. Beams **14**, 114001 (2011)

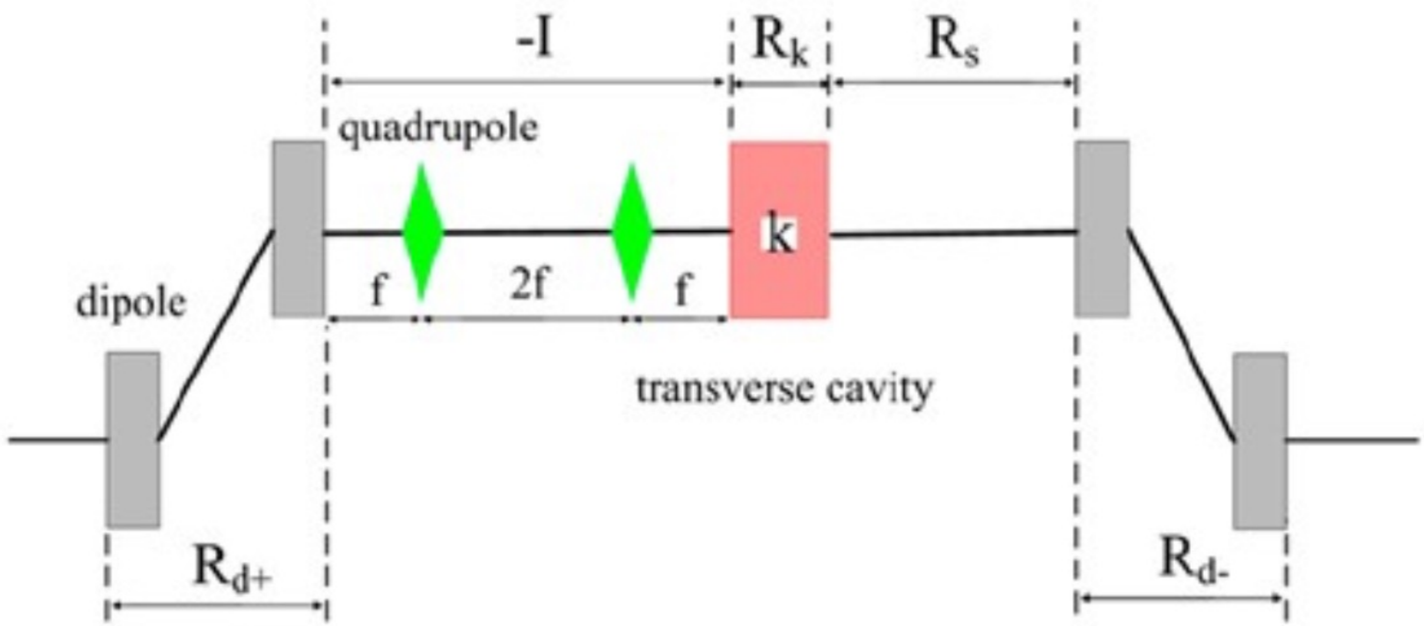
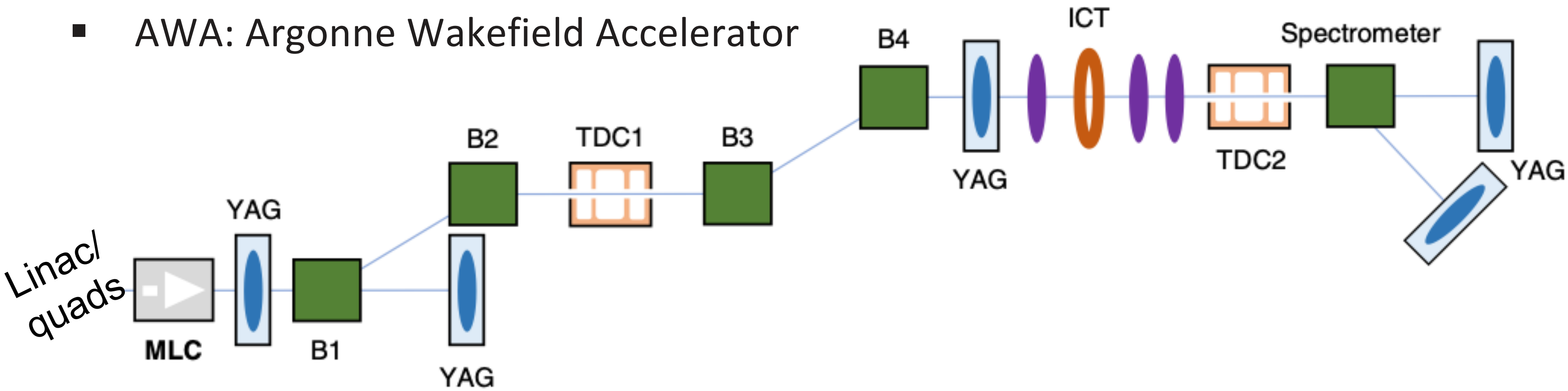


FIG. 2. A chicane-type exact EEX beam line. Two quadrupoles (green diamonds) are put upstream of the transverse cavity to reverse the dispersion.

- Reversing dispersion before the TM cavity allows you to flip the second dogleg to make a chicane
  - More transversely compact emittance exchange

AWA Longitudinal Shaping with EEX (2023)

- Majernik et al., PHYS. REV. ACCEL. BEAMS 26, 022801 (2023)
- AWA: Argonne Wakefield Accelerator



- Shape beam transversely using multi-leaf collimator
  - Near-arbitrary transverse beam shapes: EEX converts to longitudinal
  - Longitudinal pulse shaping is important for e.g. FELs, wakefield accelerators