CHAPTER 8
CONCLUDING REMARKS

In this dissertation we have described one and two-dimensional nonlinear resonances in an accelerator context and examined how motion within a one-dimensional resonance may be perturbed by tune and beta function modulation. Tune modulation was used in an experiment, E778, to measure a quantity, the island tune Q_{I}, associated with the strength of a one-dimensional resonance, showing excellent agreement with theory and simulation. This modulation also drives a nonuniversal instability, modulational diffusion, in weakly coupled systems, and this was investigated for a simple operational model of the Fermilab Tevatron collider.

Nonlinearities are always present in any accelerator, either deliberately installed or due to imperfections in magnetic field quality. A simple class of nonlinear perturbations, one-dimensional resonances, were the primary focus of this dissertation. They were shown to create structures, "resonance islands", within phase space, and capture of particles within these islands was demonstrated through simulation and experiment, creating coherent motion or "persistent signals" oscillating exactly at the resonant tune. Motion within these resonance islands was examined in a discrete Hamiltonian formalism and shown to be equivalent to that of a free pendulum. This motion was also parameterized by three quantities: the island tune, the phase space amplitude of the resonance and the resonance amplitude width.

A stability model was developed for particles oscillating close to the resonance island fixed point under the influence of tune modulation and beta modulation. This model predicts boundaries for the phase-locked stability of this motion that depend only on the island tune Q_{I} and the modulation strength and frequency. Comparison of the two showed that for realistic operational parameters the effects of tune modulation are much stronger than those of beta modulation, and both were compared to a simple robust simulation program to show excellent agreement for a particular resonance of interest.

The procedure and results of a tune modulation experiment, a portion of experiment E778 at Fermilab, were also described. The behavior of particles trapped in a nonlinear one-dimensional resonance, the $5Q_x$ resonance at the horizontal tune $Q_x = 20.40$, was systematically examined under the influence of controlled tune modulation for two distinct nonlinear configurations and three different horizontal island amplitudes. For one particular case of sextupole configuration and island amplitudes a detailed analysis of the response of the persistent signal at high frequencies agreed with the one-dimensional tune modulation model.

Frequency domain analysis was shown to be a useful tool for investigation of one-dimensional persistent signals, requiring no scaling with initial beam intensity or normalization of the beam position monitor measurement. This method has several advantages over time domain analysis, because tune modulation parameters and the island tune are all natural frequency domain variables characterizing this system. Because the tune modulation frequency can be finely controlled, this method may also allow investigation of the frequency (or particle) distribution within the nonlinear resonance island from examination of the rate of captured signal loss as the modulation frequency is increased.

The unperturbed single-resonance model was extended to two transverse dimensions, and a first order nonlinear model with two crossing resonances was examined. Simulation showed the existence of two-dimensional coherent motion and resonance islands at a particular set of base tunes corresponding to the point where the resonances under investigation crossed. The corresponding theoretical model predicted fixed point locations and phases accurately, but did not reproduce island tunes even at small nonlinear strengths. This indicates that the simple two-resonance model breaks down at the points in phase space where these resonances cross. Although persistent signals are present here, it is more likely that valuable information about the strengths of two-dimensional resonances will be obtained from two-dimensional smear, which is a qualitative measure of how the resonance distorts nearby nonresonant phase space.

An understanding of the effects of modulations leads to characteristic timescales of motion that are much larger than those typically encountered in the commissioning of many storage rings and colliders. Modulational diffusion is a previously uninvestigated amplitude growth mechanism, and it is shown here to cause luminosity loss over timescales of minutes using realistic operational parameters for the Tevatron. These timescales are long enough to prove difficult to diagnose yet short enough to significantly impact the luminosity lifetime of a collider, reducing the effectiveness of this machine in meeting experimental goals.

The character of amplitude growth due to modulational diffusion was shown to be different than that predicted by classical models, with exponential growth instead of root-time. Although not shown here, this is a general characteristic of systems where the resonance strength depends on particle amplitude, creating a feedback of this strength on the amplitude growth it creates. Modulational diffusion is not universal — there are limits that can be prescribed, corresponding to the tune modulation diagram, which preclude the existence of this mechanism in operational circumstance. These conditions can provide useful design limits for future colliders, including limits on power supply ripple and synchrotron frequency.

Future avenues of investigation and research include examination of the effects of tune modulation on two-dimensional nonlinear resonances. Though this is a more realistic scenario, the coupled nature of the motion and the complexity of examining multi-resonance systems make such an examination difficult. Since tune modulation exists in both planes of motion, it is feasible that such a mechanism could create a web of weak overlapping resonances in the tune plane, and that regions where modulational diffusion exists could be characterized.