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NONLINEAR RESONANCE ISLANDS AND MODULATIONAL
EFFECTS IN A PROTON SYNCHROTRON

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ABSTRACT

NONLINEAR RESONANCE ISLANDS AND MODULATIONAL EFFECTS IN A PROTON SYNCHROTRON

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We examine both one-dimensional and two-dimensional nonlinear resonance islands created in the transverse phase space of a proton synchrotron by nonlinear magnets. We also examine application of the theoretical framework constructed to the phenomenon of modulational diffusion in a collider model of the Fermilab Tevatron.

For the one-dimensional resonance island system, we examine the effects of two types of modulational perturbations on the stability of these resonance islands: tune modulation and beta function modulation. Hamiltonian models are presented which predict stability boundaries that depend on only three parameters: the strength and frequency of the modulation and the frequency of small oscillations inside the resonance island. These models are compared to particle tracking with excellent agreement. The tune modulation model is also successfully tested in experiment, where frequency domain analysis coupled with tune modulation is demonstrated to be useful in measuring the strength of a nonlinear resonance.

Nonlinear resonance islands are also examined in two transverse dimensions in the presence of coupling and linearly independent crossing resonances. We present a first-order Hamiltonian model which predicts fixed point locations, but does not reproduce small oscillation frequencies seen in tracking; therefore in this circumstance such a model is inadequate. Particle tracking is presented which shows evidence of two-dimensional persistent signals, and we make suggestions on methods for observing such signals in future experiment.

Finally, we apply the tune modulation stability diagram to the explicitly two-dimensional phenomenon of modulational diffusion in the Fermilab Tevatron with beam-beam kicks as the source of nonlinearity. We find that the amplitude growth created by this mechanism in simulation is exponential rather than root-time as predicted by modulational diffusion models. Finally, we comment upon the luminosity and lifetime limitations such a mechanism implies in a proton storage ring.

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