Introduction To Accelerator Physics Homework 2

Due date: Tuesday January 30, 2018

(Need help? Email `satogata at jlab.org`)

1 Cyclotron Motion

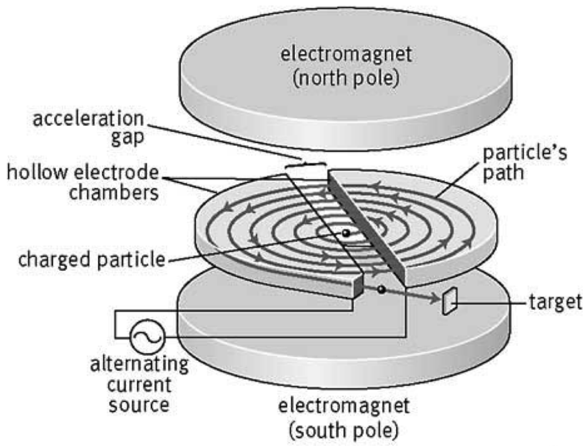

Figure 1: A schematic diagram of a cyclotron.

Consider a cyclotron pictured in Fig. **??** and as shown in class. Assume that the electromagnet forms a constant vertical magnetic field of B_y =1 T, and we inject protons with *kinetic* energy of 10 kV.

(a) (3 points) If we have the AC current source turned off so there is no acceleration, what is the radius of proton oscillations in this magnetic field?

(b) (3 points) Assume the alternating current (AC) source provides a maximum voltage between the electrode chambers (dees) of 100 kV. What frequency does this source need to be set to so the circulating protons see this full voltage every time they cross an acceleration gap?

(c) (2 points) How many revolutions ("turns") must the protons make to exit the cyclotron with a final *kinetic* energy of 1 MeV? Assume the proton stays nonrelativistic throughout, a good approximation where the final kinetic energy is much less than the rest mass energy of the proton.

(d) (2 points) At what radius would these 1 MeV protons be extracted?

(e) (5 points) Assume the protons just before they get their first acceleration are extracted immediately after the last acceleration that pushes them over the 1 MeV threshold. Calculate the total path length that a proton traverses through this cyclotron, and the total traversal time (as seen in the lab frame).

(Flip the page...)

2 The Betatron

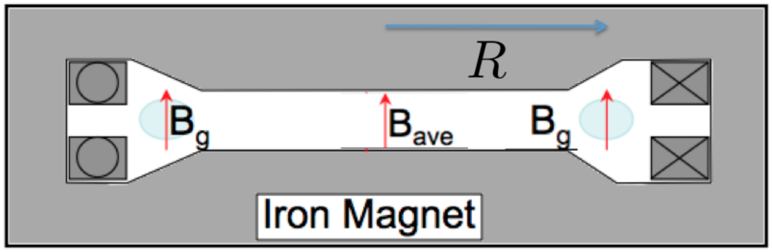

Figure 2: A cross section of a betatron. The betatron is cylindrically symmetric around the center vertical axis. The outer gray area is made of iron, while the darker rectangular regions are conductors that carry AC current clockwise or counter-clockwise.

Consider a betatron pictured in Fig. **??** and as briefly shown in class. Betatrons were originally used to accelerate electrons and illustrate our first exposure to *weak focusing*. Electrons move around the betatron in a circular path of constant radius R.

(a) (2 points) Assume that a constant current is sent through the (circular) conductors. What is magnetic flux ϕ through the circle of radius R in terms of the (spatial, not time) average magnetic field B_{ave}?

(b) (3 points) Now assume ϕ varies sinusoidally in time. Apply Faraday's law,

$$V = -\frac{d\phi}{dt}$$

to find the accelerating force that the electrons experience in terms of dB_{ave}/dt. Note that this is positive half the time and negative half the time, so at least half of the AC current cycle cannot be used to accelerate the electrons.

(c) (4 points) Note that the iron is shaped to lower the magnetic field B_g at radius R where the electrons travel. For the electrons to move in a circle of constant radius R, we need $p = eB_gR$. Use this to demonstrate that the B_g and B_{ave} must be related by

$$B_{\text{ave}} = 2B_g$$

(d) (6 points) Consider an electron betatron with a maximum B_{ave}=0.75 T and R =30 cm. Find

- the maximum electron momentum,
- the maximum electron kinetic energy (is it relativistic?),
- the required current (or ϕ) AC frequency,
- and the total number of revolutions that the electron makes during acceleration.